

Ime i prezime *ili* Šifra: _____

Broj bodova: _____

Natjecanje u integriranju *Veliki integrator*, Pismeni dio, 21. 5. 2012.

Izračunajte svaki od sljedećih integrala. Zaokružite slovo ispred točnog odgovora.

1. $\int_9^{64} \frac{1}{\sqrt{1+\sqrt{x}}} dx$
 (A) $\frac{8}{3}$ (B) $\frac{64}{3}$ (C) $\frac{8}{9}$ (D) $\frac{64}{9}$ (E) $\frac{10}{11}$
2. $\int_0^{\frac{\pi}{3}} \frac{\sin \varphi}{\cos^2 \varphi} d\varphi$
 (A) 1 (B) 2 (C) 3 (D) $\frac{\pi}{2}$ (E) $\frac{\pi}{3}$
3. $\int (\operatorname{tg}^3 x + \operatorname{tg}^5 x) dx$
 (A) $\frac{1}{4}\operatorname{tg}^4 x + C$ (B) $\operatorname{tg}^4 x + \operatorname{tg}^6 x + C$ (C) $\frac{1}{4}\operatorname{tg}^4 x + \frac{1}{6}\operatorname{tg}^6 x + C$ (D) $\frac{\operatorname{tg}^2 x(3+5\operatorname{tg}^2 x)}{\cos^2 x} + C$ (E) $\frac{\sin x}{\cos^2 x} + C$
4. $\int_0^1 x^{2012} \ln x dx$
 (A) $-\frac{1}{2011^2}$ (B) $\frac{1}{2011^2}$ (C) $-\frac{1}{2012^2}$ (D) $\frac{1}{2012^2}$ (E) $-\frac{1}{2013^2}$
5. $\int_2^3 \frac{t dt}{(t^2 + 1)^2}$
 (A) $\frac{1}{5}$ (B) $\frac{1}{10}$ (C) $\frac{1}{15}$ (D) $\frac{1}{20}$ (E) $\frac{1}{25}$
6. $\int \frac{|\cos x + \sin x|}{\sqrt{1 + \sin(2x)}} dx$
 (A) $x + C$ (B) $\sin x + C$ (C) $\cos x + C$ (D) $\operatorname{tg} x + C$ (E) $\operatorname{ctg} x + C$
7. $\int_0^1 x^x dx$
 (A) $-0.78343051 \dots$ (B) $0.78343051 \dots$ (C) $1.78343051 \dots$ (D) $100.78343051 \dots$ (E) $+\infty$
8. $\int_0^1 \frac{1 + 3\sqrt{x} + 3\sqrt{x+1}}{1 + \sqrt{x} + \sqrt{x+1}} dx$
 (A) $1 + \ln 2$ (B) $\sqrt{2} + \ln(1 + \sqrt{2})$ (C) $\sqrt{3} + \ln(1 + \sqrt{3})$ (D) $2 + \ln 3$ (E) $\sqrt{5} + \ln(1 + \sqrt{5})$
9. $\int_0^{36} [\sqrt{x}]^2 dx$
 (A) 501 (B) 502 (C) 503 (D) 504 (E) 505
10. $\int_1^2 \textit{analiza da}$
 (A) $\frac{13}{2}\textit{bonn}$ (B) $\frac{14}{3}\textit{graz}$ (C) $\frac{15}{4}\textit{linz}$ (D) $\frac{16}{5}\textit{wien}$ (E) $\frac{17}{6}\textit{zagreb}$

Veliki integrator, Pismeni dio, RJEŠENJA, 21. 5. 2012.

1. (B)
2. (A)
3. (A)
4. (E)
5. (D)
6. (A)
7. (B)
8. (B)
9. (E)
10. (C)

Natjecanje u integriranju *Veliki integrator*, Dio na ploči, 21. 5. 2012.

1.

$$\int 8 \sin^4(x/2) dx$$

2.

$$\int \frac{x(x \operatorname{tg} x + 2)}{\cos x} dx$$

3.

$$\int \frac{1 - \ln x}{x^2} dx$$

4.

$$\int \frac{x \cos x - \sin x}{(x - \sin x)^2} dx$$

5.

$$\int e^{e^x + x + e^{e^x}} dx$$

6.

$$\int \frac{dx}{(1 - x^2)^{3/2}}$$

7.

$$\int \frac{1}{x - \sqrt{x}} dx$$

8.

$$\int \frac{e^{1/x}}{x^3} dx$$

9.

$$\int 5x\sqrt{x-3} dx$$

10.

$$\int \cos\sqrt{x} dx$$

11.

$$\int x^3(x^2 + 1)^5 dx$$

12.

$$\int \frac{dx}{x \ln x}$$

13.

$$\int \sin^3 x dx$$

14.

$$\int e^{e^x+x} dx$$

15.

$$\int \cos(\sin(\sin x)) \cos(\sin x) \cos x dx$$

16.

$$\int_0^1 x(1-x)^{19} dx$$

17.

$$\int \frac{dx}{1 + e^x}$$

18.

$$\int \frac{x \operatorname{ch} x}{\operatorname{sh}^2 x} dx$$

19.

$$\int \frac{x^{-1/2}}{1 + x^{1/3}} dx$$

Veliki integrator, Dio na ploči, RJEŠENJA, 21. 5. 2012.

1.
$$\int 8 \sin^4(x/2) dx = 3x - 4 \sin x + \frac{1}{2} \sin 2x + C$$

2.
$$\int \frac{x(x \operatorname{tg} x + 2)}{\cos x} dx = \frac{x}{\cos^2 x} + C$$

3.
$$\int \frac{1 - \ln x}{x^2} dx = \frac{\ln x}{x} + C$$

4.
$$\int \frac{x \cos x - \sin x}{(x - \sin x)^2} dx = \frac{x + \sin x}{2x - 2 \sin x} + C$$

5.
$$\int e^{e^x + x + e^{e^x}} dx = e^{e^{e^x}} + C$$

6.
$$\int \frac{dx}{(1 - x^2)^{3/2}} = \frac{x}{\sqrt{1 - x^2}} + C$$

7.
$$\int \frac{1}{x - \sqrt{x}} dx = 2 \ln |\sqrt{x} - 1| + C$$

8.
$$\int \frac{e^{1/x}}{x^3} dx = e^{1/x} \left(1 - \frac{1}{x}\right) + C$$

9.
$$\int 5x\sqrt{x-3} dx = 2(x-3)^{5/2} + 10(x-3)^{3/2} + C$$

10.
$$\int \cos \sqrt{x} dx = 2(\sqrt{x} \sin \sqrt{x} + \cos \sqrt{x}) + C$$

11.
$$\int x^3(x^2 + 1)^5 dx = \frac{1}{14}(x^2 + 1)^7 - \frac{1}{12}(x^2 + 1)^6 + C$$

12.
$$\int \frac{dx}{x \ln x} = \ln |\ln x| + C$$

13.
$$\int \sin^3 x dx = \frac{1}{3} \cos^3 x - \cos x + C$$

14.
$$\int e^{e^x + x} dx = e^{e^x} + C$$

15.

$$\int \cos(\sin(\sin x)) \cos(\sin x) \cos x \, dx = \sin(\sin(\sin x)) + C$$

16.

$$\int_0^1 x(1-x)^{19} \, dx = \frac{1}{420}$$

17.

$$\int \frac{dx}{1+e^x} = x - \ln(e^x + 1) + C$$

18.

$$\int \frac{x \operatorname{ch} x}{\operatorname{sh}^2 x} \, dx = -\frac{x}{\operatorname{sh} x} + \ln(\operatorname{sh} \frac{x}{2}) - \ln(\operatorname{ch} \frac{x}{2}) + C$$

19.

$$\int \frac{x^{-1/2}}{1+x^{1/3}} \, dx = 6x^{1/6} - 6 \arctan(x^{1/6}) + C$$