$pV = \frac{m}{M} RT$

$p = \frac{\rho}{M} RT$

zrak

78% N_2	21.84
21% O_2	6.72
1% Ar	0.40
	28.96 g/mol = 0.029 kg/mol !

$\rho_z = \frac{p_0 M_z}{R \cdot T} = \frac{101325 \cdot 0.029}{8.314 \cdot 273} = 1.295 \text{ kg/m}^3$

⑦ Gustoća zraka iznosi 1.3 kg/m³ pri određenom tlaku i temperaturi. Odredite gustoću slijedećih plinova pri istim uvjetima

	M	ρ
H_2	2	0.09
He	4	0.18
O_2	32	1.43
Ne	20.2	0.90
CO_2	44	1.97
CO	28	1.26
CH_4	16	0.72

kg/m³

$pM = \rho RT$

$\frac{\rho_1}{M_1} = \frac{\rho_2}{M_2} \left(= \frac{p}{RT}\right)$

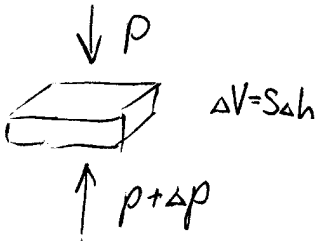
$\rho_{PLINA} = \rho_{ZRAKA} \cdot \frac{M_{PLINA}}{M_{ZRAKA}}$

(8) Pri izobarnom zagrijavanju dušika sa 10°C na 30°C gustoća se smanjila za 0.11 kg/m³. Koliki je bio (i ostao) tlak?

$$\rho_1 = \frac{pM}{R}\frac{1}{T_1}$$
$$\rho_2 = \frac{pM}{R}\frac{1}{T_2}$$

$$0.11 = \rho_1 - \rho_2 = \frac{pM}{R}\left(\frac{1}{283} - \frac{1}{303}\right)$$
$$0.91454 = p \cdot 0.028 \cdot 0.00023324$$
$$p = 140\,037\ Pa$$

(9) Odredite ovisnost atmosferskog tlaka o visini u izotermnom ($T = 273K$) idealnom plinu srednje molekulske mase 29 g/mol. Uzeti $g = 10\ m/s^2$



$$\Delta p = \frac{\Delta F}{S} = \frac{\Delta m g}{S} = -\frac{\rho g \Delta V}{S} = -\rho g \Delta h$$
$$p = \frac{\rho RT}{M} \Rightarrow \frac{\Delta p}{\Delta h} = -\rho g = -p \cdot \frac{Mg}{RT}$$
$$p = p_0 e^{-\frac{Mgh}{RT}} = p_0 e^{-\frac{h}{h_0}}$$
$$h_0 = \frac{RT}{Mg} = 7827\ m$$
$$\frac{dp}{dh} = p \cdot \left(-\frac{Mg}{RT}\right) \checkmark$$

Koliki je tlak na 3000 m nadmorske visine?

$$p(3000m) = p_0 e^{-\frac{h}{h_0}} = 101325\, e^{-\frac{3000}{7827}} = 69\,063\ Pa\ (68.16\%\ p_0)$$

(10) Odredite gustoću mješavine 30% kisika i 70% helija (volumni u.) pri tlaku 10^6 Pa i temperaturi 290 K

$M(O_2) = 32$ g/mol
$M(He) = 4$ g/mol

$$\bar{M} = 0.3 \cdot 32 + 0.7 \cdot 4 = 12.4\ g/mol = 0.0124\ kg/mol$$
$$pM = \rho RT$$
$$12\,400 = \rho \cdot 2411$$
$$\rho = 5.14\ kg/m^3$$

Valovi: Zvuk, Svjetlost, Valovi na vodi, titrajuće žice

(11) Brzina zvuka u idealnom plinu računa se izrazom $v^2 = \frac{\gamma p}{\rho} = \frac{\gamma kT}{m} = \frac{\gamma RT}{M}$. Odredite brzinu zvuka na Zemlji (zrak, $T = 0°C$) i Marsu (CO_2, $T = -50°C$).

$$v_Z^2 = \frac{\gamma RT}{M} = \frac{1.4 \cdot 8.314 \cdot 273}{0.029} \Rightarrow v_Z = 331 \text{ m/s}$$

$$\gamma = \frac{7 \cdot 0.99 + 5 \cdot 0.01}{5 \cdot 0.99 + 3 \cdot 0.01} = \frac{6.98}{4.98} = 1.4016$$

$$v_M^2 = \frac{1.4 \cdot 8.314 \cdot 223}{0.044} \Rightarrow v_M = 242.9 \text{ m/s}$$

---

(12) Žica mase 10g duljine 1m učvršćena je na oba kraja i nategnuta silom od 100N

a) Odredite brzinu širenja valova žicom

$$F = \frac{mv^2}{\ell} \Rightarrow v = \sqrt{\frac{F\ell}{m}} = 100 \text{ m/s}$$

b) Odredite moguće frekvencije titranja

$\lambda_1 = 2m$, $\lambda_2 = 1m$, $\lambda_3 = \frac{2}{3}m$, ⋮

$$\lambda_n = \frac{2\ell}{n}, \quad f_n = \frac{v_0}{\lambda_n} = n \cdot 50 \text{ Hz}$$

$f_1 = 50$ Hz
$f_2 = 100$ Hz
$f_3 = 150$ Hz
⋮

c) Kolika je valna duljina osnovnog tona u zraku? ($v = 331$ m/s)

$$\lambda = \frac{v_z}{f_1} = 6.62 \text{ m}$$

---

(13) Zemljinom se korom longitudinalni valovi šire brzinom 6 km/s, a transverzalni 3.5 km/s. Odredite udaljenost epicentra potresa od geofizičke postaje ako su longitudinalni valovi stigli 15s prije transverzalnih

$$\left.\begin{aligned} d = v_1 t_1 &= v_2 t_2 \\ t_1 + 15s &= t_2 \end{aligned}\right\} \quad 6t_2 - 6 \cdot 15 = 3.5 t_2$$

$$2.5 t_2 = 90$$

$$t_2 = 36s$$

$$t_1 = 21s$$

$$d = 6 \cdot 21 = 3.5 \cdot 36 = 126 \text{ km}$$

(14) Ovisnost brzine valova na vodi o valnoj duljini zadana je relacijom:

$$v(\lambda) = \sqrt{\frac{g\lambda}{2\pi} + \frac{2\pi\gamma}{\rho\lambda}}$$

$g = 9.81\ m/s^2$
$\rho = 1000\ kg/m^3$
$\gamma = 07\ N/m$ (napetost površ.)

a) Odredite brzinu valova valne duljine 2 m

$$v = \sqrt{\frac{9.81 \cdot 2}{2 \cdot 3.14} + \frac{2 \cdot 3.14 \cdot 07}{1000 \cdot 2}} = 1.768\ m/s$$

b) Odredite najmanju brzinu valova i valnu duljinu koju imaju valovi koji putuju tom brzinom.

$$v^2 = \frac{g\lambda}{2\pi} + \frac{2\pi\gamma}{\rho\lambda}$$

$$\frac{d}{d\lambda} v^2 = \frac{g}{2\pi} + \frac{2\pi\gamma}{\rho} \cdot \left(-\frac{1}{\lambda^2}\right) = 0$$

0.053075535

$$\frac{g}{2\pi} = \frac{2\pi\gamma}{\rho\lambda_m^2} \Rightarrow \lambda_m^2 = \frac{4\pi^2\gamma}{\rho g} \Rightarrow \lambda_m = 0.053\ m = 5.3\ cm$$

$$v_m = \sqrt{0.08279 + 0.08258} = 0.4071\ m/s$$

---

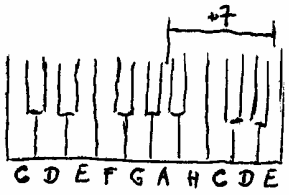
(15) Oktava se sastoji od 12 polutonova koji su razmaknuti tako da je omjer frekvencija dva susjedna polutona konstantan

a) koliki je taj omjer?

$$f_0, xf_0, x^2f_0, x^3f_0 \ldots x^{12}f_0 = 2f_0 \Rightarrow x = \sqrt[12]{2} = 1.0594631$$

b) ako tonu A odgovara frekvencija 440 Hz, kolika je frekvencija tona E? (razlika je +7 ili -5 polutonova)

$$f_E = f_A \cdot x^7 = 440\ Hz \cdot 2^{7/12} = 659.255\ Hz$$

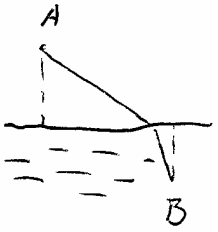


$$\mu_0 = 4\pi \cdot 10^{-7}\ \frac{T \cdot m}{A}$$

$$\varepsilon_0 = 8.8541 \cdot 10^{-12}\ \frac{Nm^2}{C^2}$$

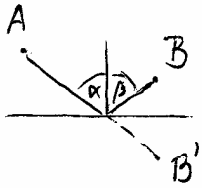
Svjetlost: $v^2 = c^2 = \frac{1}{\varepsilon_0 \mu_0}$ $\quad c = 299\,792\,458\ m/s$

Fermat-ov princip najbržeg puta:

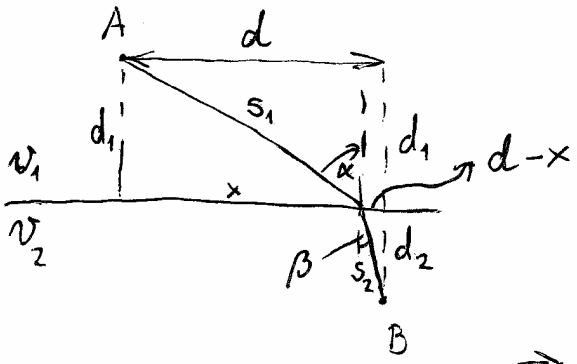


Zraka svjetlosti koja prolazi točkama A i B između njih će proći najbržim putem

a) Pravocrtno gibanje svjetlosti
b) Zakon refleksije: $\beta = \alpha$
c) Zakon loma $\frac{\sin\alpha}{\sin\beta} = n$



(16) Izvedite Snellov zakon loma $\left(\frac{\sin\alpha}{\sin\beta} = n\right)$ iz Fermat-ovog principa



$$t = \frac{s_1}{v_1} + \frac{s_2}{v_2} = \frac{\sqrt{d_1^2 + x^2}}{v_1} + \frac{\sqrt{d_2^2 + (d-x)^2}}{v_2}$$

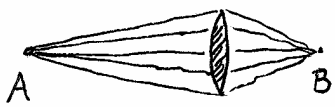
$$\frac{dt}{dx} = 0 = \frac{1}{v_1}\frac{2x}{2\sqrt{d_1^2+x^2}} - \frac{2(d-x)}{2\sqrt{d_2^2+(d-x)^2}}$$

$$\frac{1}{v_1}\sin\alpha - \frac{1}{v_2}\sin\beta = 0$$

$$\Rightarrow \frac{\sin\alpha}{\sin\beta} = \frac{v_1}{v_2} = n$$

- Ravno ogledalo
- Prizma
- Ravno staklo

Fokusiranje:



Da bismo postigli fokusiranje snopa zraka iz točke A u točki B, svi putovi iz A u B moraju biti jednako brzi.

$$j = \frac{1}{f}, \quad j \approx \frac{n-1}{R} \text{ (tanke leće)}$$

Jačina je aditivna veličina. Dimenzija je $dpt = m^{-1}$

(17) Izvor svjetlosti udaljen je 2 metra od tanke plankonveksne leće radijusa zakrivljenosti 30 cm. Oštra slika nastaje na 80 cm udaljenosti od leće. Odredite indeks loma materijala leće. Koliko je uvećanje?

$$j = \frac{1}{2} + \frac{1}{0.8} = 1.75\,dpt \qquad j = (n-1)\cdot\frac{1}{R}$$

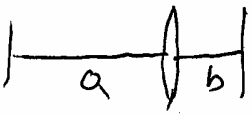
$$n - 1 = 1.75 \cdot 0.3 = 0.525$$

$$\boxed{n = 1.525}$$

$$\boxed{m = -\frac{b}{a} = -\frac{0.8}{2} = -0.4}$$

---

(18) Udaljenost predmeta i slike je 60 cm. Jačina leće između njih je +8 dpt. Kolika je udaljenost leće i predmeta?



$$\left.\begin{aligned} a + b &= 0.6\ m \\ \frac{1}{a} + \frac{1}{b} &= 8\ dpt \end{aligned}\right\} \quad \frac{1}{a} + \frac{1}{0.6-a} = 8$$

$$0.6a + a = 8a(0.6-a) \quad /:8$$

$$a^2 - 0.6a + 0.075 = 0$$

$a_1 = 0.4225\ m \quad b_1 = 0.1775\ m$

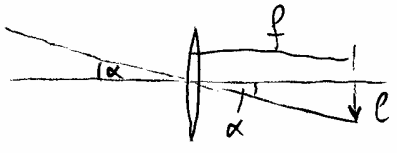
$a_2 = 0.1775\ m \quad b_2 = 0.4225\ m$

---

(19) Odredite položaj i žarišnu daljinu leće pomoću koje dobijemo 11× uvećanu sliku na zastoru udaljenom 6 m

$$\left.\begin{aligned} \frac{b}{a} &= 11 \\ a + b &= 6 \end{aligned}\right\} \Rightarrow \left.\begin{aligned} a &= 0.5\ m \\ b &= 5.5\ m \end{aligned}\right\} \quad j = \frac{1}{a} + \frac{1}{b} = 2.18\,dpt,\ f = \underline{45.83\ cm}$$

(20) Koliko je velika realna slika Sunca koju stvara konvergentna leća žarišne daljine 50 cm? Prividni promjer Sunca je 0°32'.



$$\sin\alpha = \frac{\ell}{f} \Rightarrow \ell = f\sin\alpha$$

$$= 50\,cm \cdot 0.0093083$$

$$= \underline{0.4654\ cm}$$

Vježbe Fizika 2

Temperatura i toplina
Specifični toplinski kapacitet
Latentna toplina taljenja i isparavanja
Talište i vrelište
Entropija $\Delta S = \frac{\Delta Q}{T}$, (Entalpija $\Delta H = T\Delta S$)
Voda, led, vodena para

$\Delta T = 20°C = 20K$
$T_1 = 0°C = 273K$
$T_2 = 20°C = 293K$

$\Delta Q = mc\Delta T$ - grijanje - hlađenje
$\Delta Q = m\lambda$ - taljenje, očvršćivanje
$\Delta Q = mr$ - isparavanje, kondenz.

$T_s = 0°C$, $\lambda = 330\,000\ J/kg$
$T_p = 100°C$, $r = 2\,260\,000\ J/kg$
$c_L = 2100\ J/kg\,K$
$c_v = 4190\ J/kg\,K$

tvar	c (J/kgK)
Fe	460
Cu	380
Al	920
C_2H_5OH	2500

(1) U kalorimetru se nalazi 5 dcl vode, temperature 30°C. Ubacimo 100g leda temperature 0°C. Odredite ravnotežnu temperaturu. Gubitke topline zanemarite.

taljenje leda: $Q_1 = m\lambda = 0.1kg \cdot 330\,000\ J/kg = 33\,000\ J$
hlađenje vode 30-0°C: $Q_2 = m_v c_v \Delta T = 0.5 \cdot 4190 \cdot 30 = 62\,850\ J$
$Q_1 < Q_2 \Rightarrow$ sav led se otopi i zagrije do ravnotežne temperature τ

led: $Q_L = m_L\lambda + m_L c_v (\tau - 0°C)$
voda: $Q_v = m_v c_v (30°C - \tau)$

$Q_L = Q_v$
$33000 + 419\tau = 2095(30 - \tau)$
$\tau = \frac{29\,850}{2514} = 11.87°C$

(2) U kalorimetru se nalazi 5 dcl vode, temperature 30°C. Ubacimo 250g leda temperature 0°C. Odredite količinu neotopljenog leda. Gubitke topline zanemarite.

$Q_1 = m_L\lambda = 82\,500\ J$
$Q_2 = m_v c_v \Delta T = 62\,850\ J$
$Q_1 > Q_2$, $\Delta Q = 19\,650\ J$
$\Delta m = \frac{\Delta Q}{\lambda} = 0.06\ kg = 60g$

③ U kalorimetru se nalazi 1.5 kg vode temperature 18°C. Ubacimo dvije aluminijske kugle mase po 0.8 kg, prva temperature 85°C, a druga 35°C. Odredite ravnotežnu temperaturu. $c_{Al} = 920\ J/kg\,K$

$T_1 = 85°C$ $\quad T_2 = 35°C$ $\quad T_3 = 18°C$

$m_1 = 0.8\ kg$ $\quad m_2 = 0.8\ kg$ $\quad m_3 = 1.5\ kg$

$c_1 = 920\ J/kg\,K$ $\quad c_2 = 920\ J/kg\,K$ $\quad c_3 = 4190\ J/kg\,K$

$\Rightarrow$ ravnotežna temperatura je τ.

$$m_1 c_1 (T_1 - \tau) + m_2 c_2 (T_2 - \tau) = m_3 c_3 (\tau - T_3)$$

$$m_1 c_1 T_1 + m_2 c_2 T_2 + m_3 c_3 T_3 = \tau (m_1 c_1 + m_2 c_2 + m_3 c_3)$$

$$\tau = \frac{736 \cdot 85 + 736 \cdot 35 + 6285 \cdot 18}{736 + 736 + 6285} = \frac{201\,450}{7\,757} = 25.97\,°C$$

④ U kalorimetru pomiješamo 0.2 kg leda temperature 0°C i 0.4 kg vode temperature 60°C. Odredite ravnotežnu temperaturu i porast entropije.

$$\left.\begin{aligned} Q_1 &= m_L \lambda_L + m_L c_V \tau \\ Q_2 &= m_V c_V (60 - \tau) \end{aligned}\right\} \quad 66\,000 + 838\tau = 100\,560 - 1676\tau$$

$$\tau = 13.75\,°C$$

$$S_1 = \frac{m_L \lambda_L}{T_L} = \frac{66000}{273} = 241.76\ J/K$$

$$S_2 = m_L c_V \ln\left(\frac{\tau}{T_L}\right) = 838 \cdot \ln\frac{286.75}{273} = 41.18\ J/K$$

$$S_3 = m_V c_V \ln\left(\frac{\tau}{T_V}\right) = 1676 \ln\frac{286.75}{333} = -250.62\ J/K$$

$$\Delta S = S_1 + S_2 + S_3 = +32.32\ J/K$$

$$\Delta S = \frac{\Delta Q}{T}$$

$$\Delta S = mc\frac{\Delta T}{T}$$

$$S = mc \ln\frac{T_1}{T_2}$$

Vježbe

⑤ U kalorimetru pomiješamo 0.4 kg vode temperature 5°C i 1.6 kg vode temperature 22°C. Kolika je ravnotežna temperatura? Koliki je porast entropije?

$T_2 = 295\ K$
$T_1 = 278\ K$

$$\tau = \frac{m_1 c_1 T_1 + m_2 c_2 T_2}{m_1 c_1 + m_2 c_2} = \frac{m_1 T_1 + m_2 T_2}{m_1 + m_2} = \frac{0.4 \cdot 5 + 1.6 \cdot 22}{0.4 + 1.6} = 18.6\,°C = 291.6\ K$$

$$\Delta S_1 = m_1 c_1 \ln \frac{\tau}{T_1} = 0.4 \cdot 4190 \ln \frac{291.6}{278} = 80.05\ J/K$$

$$\Delta S_2 = m_2 c_2 \ln \frac{\tau}{T_2} = 1.6 \cdot 4190 \cdot \ln \frac{291.6}{295} = -77.72\ J/K$$

$$\Delta S = 2.33\ J/K$$

---

⑥ U kalorimetru se nalazi 0.8 kg leda. Ulijemo toliko vode temperature 20°C da se u ravnotežnom stanju sav led otopi (i ne zagrijava dalje). Koliko smo vode ulili? Koliki je porast entropije? Gubitke topline zanemarujemo.

$$Q_1 = Q_2 = m_1 \lambda = 0.8 \cdot 330\,000 = 264\,000\ J$$

$$m_2 = \frac{Q_2}{c_v \Delta T} = \frac{264\,000}{4190 \cdot 20} = 3.15036\ kg$$

$$\Delta S_1 = \frac{Q_1}{T} = \frac{26400}{273} = 967.033\ \frac{J}{K}$$

$$\Delta S_2 = m_2 c_2 \ln \frac{\tau}{T_2} = 13200 \ln \frac{273}{293} = -933.251\ J/K$$

$$\Delta S = \Delta S_1 + \Delta S_2 = +33.782\ J/K$$

$$pV = \frac{m}{M} RT$$

$$p = \frac{\rho}{M} RT$$

zrak

78% N_2	21.84	
21% O_2	6.72	
1% Ar	0.40	
	28.96 g/mol	= 0.029 kg/mol

$$\rho_z = \frac{p_0 M_z}{R \cdot T} = \frac{101325 \cdot 0.029}{8.314 \cdot 273} = 1.295\ kg/m^3$$

⑦ Gustoća zraka iznosi 1.3 kg/m³ pri određenom tlaku i temperaturi. Odredite gustoću slijedećih plinova pri istim uvjetima

	M	ρ	
H_2	2	0.09	kg/m³
He	4	0.18	
O_2	32	1.43	
Ne	20.2	0.90	
CO_2	44	1.97	
CO	28	1.26	
CH_4	16	0.72	

$$pM = \rho RT$$

$$\frac{\rho_1}{M_1} = \frac{\rho_2}{M_2} \left(= \frac{p}{RT}\right)$$

$$\rho_{PLINA} = \rho_{ZRAKA} \cdot \frac{M_{PLINA}}{M_{ZRAKA}}$$