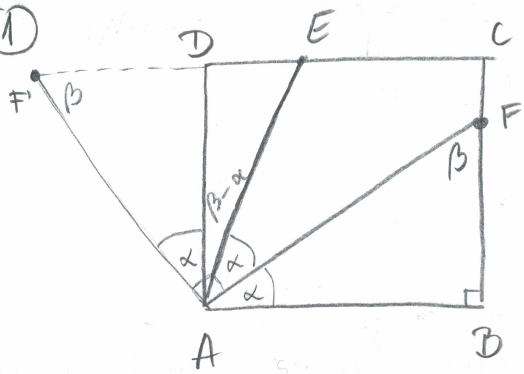
EG 3. rok 29.8.2024.- SKICE RJEŠENJA

1)



$\sphericalangle BAF = \sphericalangle FAE =: \alpha$

$\Rightarrow$ neka je r rotacija oko A za $90°$ i $F' = r(F)$

$\Rightarrow$ • $\sphericalangle FAF' = 90° \Rightarrow \sphericalangle F'AD = \alpha$

• $|AF| = |AF'|$ (rotacija)

• $|AD| = |AB|$ (kvadrat)

$\hookrightarrow$ [S-K-S] $\Rightarrow \Delta ADF' \cong \Delta ABF$

$\Rightarrow \sphericalangle F'DA = 90°$

$\Rightarrow F', D, E, C$ kolinearni

• neka je $\sphericalangle AFB = \beta$

$(\alpha + \beta = 90°)$

$\Rightarrow$ [sukladnost]

$\sphericalangle DF'A = \beta$, $\sphericalangle D = 90°$

$\sphericalangle DAE = 90° - 2\alpha = \overbrace{90° - \alpha} - \alpha = \beta - \alpha$

$\Rightarrow \sphericalangle F'AE = \alpha + (\beta - \alpha) = \beta = \sphericalangle EF'A$

$\Leftrightarrow \Delta F'AE$ jednakokračan $\Leftrightarrow |F'E| = |AE|$

$\Leftrightarrow |F'D| + |DE| = |AE|$

(sukl.) $\Leftrightarrow |BF| + |DE| = |AE|$ $\blacksquare$

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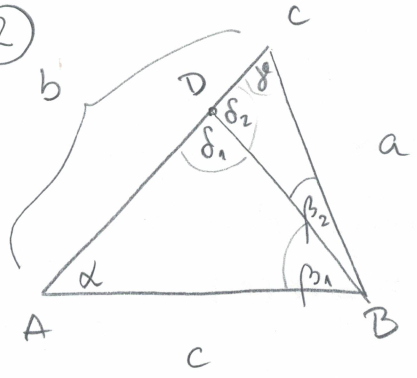

$|AD| = \frac{3}{4}|AC|$ $\left(\Rightarrow |AD| = 3|DC|, \; |DC| = \frac{1}{4}|AC| \right)$

• označimo kuteve kao na skici

$\triangle ABD$ i $\triangle DCB$ slični $\Rightarrow$ odgovarajući kutevi jednaki

$\rightarrow$ koji su odgovarajući parovi?

• očito je $\delta_1 \neq \gamma$ i $\alpha \neq \delta_2$ (inače $DB \parallel CB$, tj. $AB \parallel BD$)

1°) $\alpha = \gamma$? $\Rightarrow$ jednakokr. trokut ↯

2°) $\alpha = \delta_2$? $\Rightarrow$ $AB \parallel DB$ ↯

$\Rightarrow$ $\boxed{\alpha = \beta_2}$ $\rightarrow$ $\gamma = ?$

$\rightarrow$ ne može $\gamma = \delta_1$

$\Rightarrow$ $\boxed{\gamma = \beta_1}$ $\Rightarrow$ $\boxed{\delta_1 = \delta_2}$

$\delta_1 = \delta_2$ i $\delta_1 + \delta_2 = 180°$ $\Rightarrow$ $\delta_1 = \delta_2 = 90°$ $\Rightarrow$ $\alpha + \beta_1 = 90°$, $\gamma + \beta_2 = 90°$

$\rightarrow$ $\boxed{\beta_2 + \beta_1 = 90°} =: \beta$

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neka je $|BD| =: x$, imamo $|AD| = \frac{3}{4}b$, $|CD| = \frac{1}{4}b$

Pitagora:
$$\left.\begin{aligned} a^2 + c^2 &= b^2 \\ \left(\tfrac{3}{4}b\right)^2 + x^2 &= c^2 \\ \left(\tfrac{1}{4}b\right)^2 + x^2 &= a^2 \end{aligned}\right\} \Rightarrow (\ldots) \Rightarrow \boxed{\begin{aligned} x &= \tfrac{\sqrt{3}}{4}b, \\ a &= \tfrac{1}{2}b \\ c &= \tfrac{\sqrt{3}}{2}b \end{aligned}}$$

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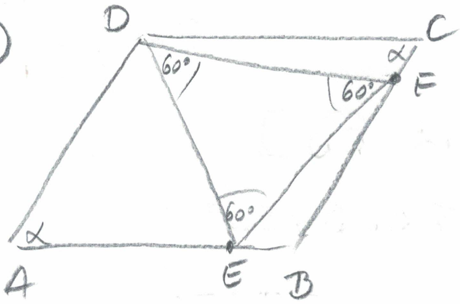
trig. pravokutnog trokuta:

$\sin\alpha = \frac{a}{b} = \frac{\frac{1}{2}b}{b} = \frac{1}{2}$ $\Rightarrow$ $\alpha = 30°$ $\rightarrow$ $\gamma = 90° - 30° = 60°$

(3)



$|EB| = \frac{1}{6}|AB|$
$|CF| = \frac{1}{6}|BC|$

prvo pretp. da je DEF jednakostr.

$\alpha := \sphericalangle DAB = \sphericalangle DCB \Rightarrow \sphericalangle ABC = 180° - \alpha$

$a := |AB| = |BC| = |CD| = |AD|$

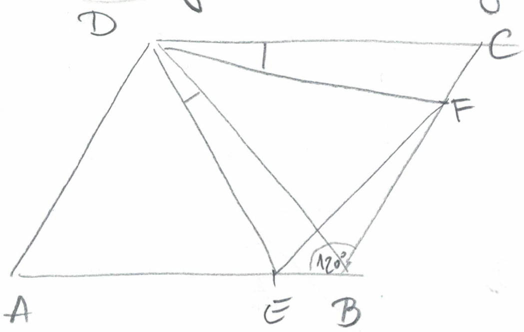
$x := |DE| = |EF| = |DF|$

$\triangle DCF \to$ kosinusov poučak $x^2 = \ldots$

$\triangle EBF \to$ — " — $x^2 = \ldots$

izjednačimo izraze za $x^2 \Rightarrow \alpha = 60°$,
$\sphericalangle CBA = 120°$

2. smjer → neka je $\sphericalangle CBA = 120°$



$|EB| = |CF| = \frac{1}{6}|AB| = \frac{1}{6}a$ (iste oznake)

$\sphericalangle CBA = 120° \Rightarrow \sphericalangle DCB = 60°$

$\Rightarrow$ $\triangle ABD$ i $\triangle BCD$ su jednakokr. (kraci su str. romba) s kutem $60°$

$\Rightarrow$ $\triangle ABD \cong \triangle BCD$ jednakostr.

↳ sukladnost jer imamo i jednakost stranica

$$\left.\begin{array}{l} |EB| = |CF| = \frac{1}{6}a \\ \sphericalangle DCB = 60° = \sphericalangle DBE \\ |BD| = |CD| = a \end{array}\right\} \quad \text{S-K-S:} \quad \triangle EBD \cong \triangle BCD$$

$\Rightarrow \sphericalangle EDB = \sphericalangle FDC \Rightarrow$
$\sphericalangle EDF = \sphericalangle EDB + \sphericalangle BDF$
$= \sphericalangle FDC + \sphericalangle BDF$
$= \sphericalangle BDC = 60°$

$\Rightarrow |DF| = |DE| \Rightarrow \triangle DEF$ jednakokračan s kutem $60°$

$\Rightarrow \triangle DEF$ jednakostr.

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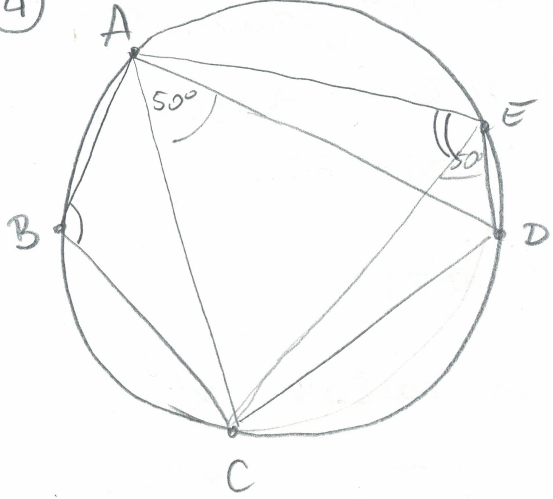

$\sphericalangle CAD = 50^\circ$

$\Rightarrow$ [obodni nad $\overset{\frown}{CD}$]

$\Rightarrow \sphericalangle CED = 50^\circ$

ABCE tetivan

$\Rightarrow \sphericalangle ABC + \sphericalangle AEC = 180^\circ$

$\Rightarrow \sphericalangle AEC = 180^\circ - \sphericalangle ABC$

$$\sphericalangle ABC + \sphericalangle AED = \underbrace{\sphericalangle ABC + \sphericalangle AEC}_{} + \underbrace{\sphericalangle CED}_{}$$

$$= 180^\circ + 50^\circ$$

$$= 230^\circ$$

2. način (upr.)

↳ raspisati jednakosti obodnih kuteva nad stranicama peterokuta

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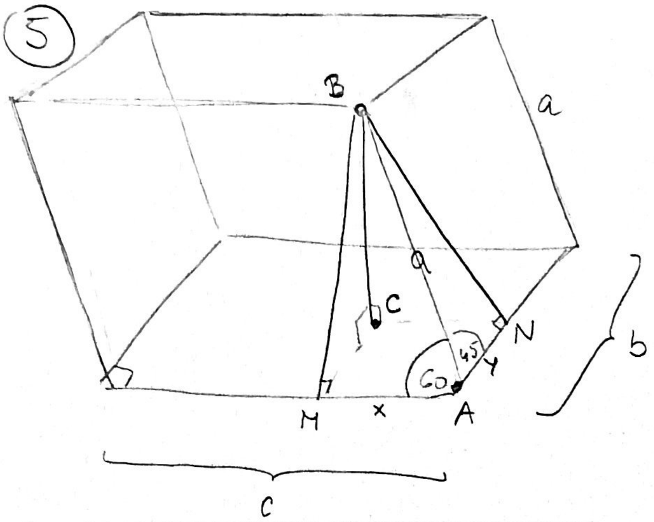
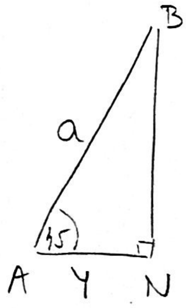
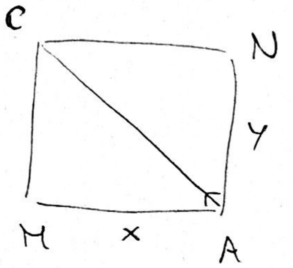




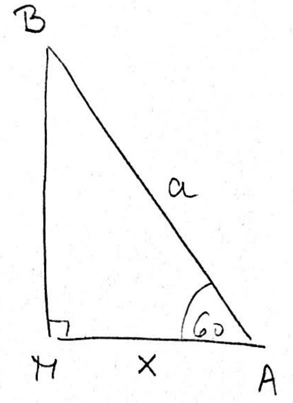
$$y = a\cos 45^\circ$$

$$y = a\frac{\sqrt{2}}{2}$$



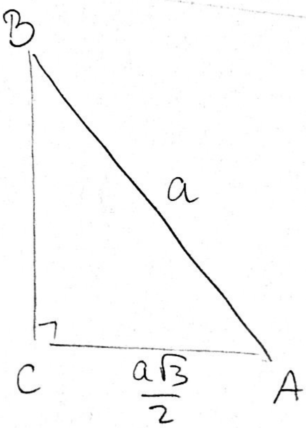
($\overline{AC}$ je projekcija brida $\overline{AB}$ na bazu dolje)

$$|AC| = \sqrt{x^2+y^2} = \sqrt{\frac{a^2}{4}+\frac{a^2\cdot 2}{4}} = \frac{a\sqrt{3}}{2}$$



$$x = a\cos 60^\circ$$

$$x = \frac{a}{2}$$



$$h = |BC| = \sqrt{a^2-\left(\frac{a\sqrt{3}}{2}\right)^2}$$

$$\boxed{h = \frac{a}{2}}$$

$B =$ (površina baze) $= b\cdot c$

$$\boxed{V = B\cdot h = \frac{abc}{2}}$$

Kao bazu uzimamo pravokutnik radi lakšeg rješavanja zadatka.

Objašnjenje oznaka na skici: C je nožište visine prizme iz vrha B spuštene na bazu (dolje), M i N su nožišta visina (okomica) iz B na bridove iz A duljine b i c.