## 7.2 Logistička regresija

↳ za $K=2$, tj. $S=\{0,1\}$, to je GLM za $\{B(1,p)\}$ familiju uz kanonsku f-ju veze $g$ (tzv. logit f-ju): za $\beta \in \mathbb{R}^p$ pretpost.

$$\log\left(\frac{p_1(x)}{\underbrace{1-p_1(x)}_{=p_0(x)}}\right) = g(p_1(x)) \stackrel{!}{=} x^T\beta, \quad x \in \mathbb{R}^p \qquad (7.6)$$

tj.

$$p_1(x) = g^{-1}(x^T\beta) \stackrel{!}{=} \frac{e^{x^T\beta}}{1+e^{x^T\beta}}, \quad x \in \mathbb{R}^p \qquad (7.7)$$

→ $\dfrac{p_1(x)}{p_0(x)}$ su "izgledi" (engl. odds), npr.

- $p_1 = \frac{2}{3} \Rightarrow p_0 = \frac{1}{3}$ i odds $= \frac{2}{1}$
- $p_1 = \frac{2}{5} \Rightarrow p_0 = \frac{3}{5}$ i odds $= \frac{2}{3}$.

Interpretacija parametara $\beta$?

↳ uz (7.6) (tj. (7.7)), za $x=(x_1,\dots,x_p) \in \mathbb{R}^p$ te

$$x' = (x_1,\dots,x_{j-1}, \boxed{x_j+1}, x_{j+1},\dots,x_p) \in \mathbb{R}^p, \text{ imamo}$$

$$\log\left(\frac{p_1(x')}{p_0(x')}\right) = \log\left(\frac{p_1(x)}{p_0(x)}\right) + \beta_j, \text{ tj.}$$

$$\frac{p_1(x')}{p_0(x')} = \frac{p_1(x)}{p_0(x)} \cdot \boxed{e^{\beta_j}} \quad \begin{cases} > 1, & \text{ako } \beta_j > 0 \\ < 1, & \text{ako } \beta_j < 0. \end{cases}$$

→ $\boxed{\text{pusenje.R}}$

### 7.2.1 Multinomijalna logistička regresija ($K \in \mathbb{N}$)

↳ pretp. da za neke $\beta^{(1)},\dots,\beta^{(K-1)} \in \mathbb{R}^p$,

$$\log\left(\frac{p_k(x)}{p_0(x)}\right) = x^T\beta^{(k)}, \quad x \in \mathbb{R}^p, \quad k=1,\dots,K-1 \qquad (7.8)$$

↳ [K-1 nezavisnih "log. regr." za $k$ u odnosu na $0$.]

**Napl** Kategorija 0 je tzv. bazna kategorija (engl. baseline).

proizvoljno!

(7.8) $\Rightarrow$ za $x, x'$ kao gore $(x' = x + e_j)$,

$$\frac{p_k(x')}{p_0(x')} = \frac{p_k(x)}{p_0(x)} \cdot e^{\beta_j^{(k)}}.$$

DZ Pokažite da je (7.8) ekvivalentno s

$$p_k(x) = \frac{e^{x^T \beta^{(k)}}}{1 + \sum_{i=1}^{K-1} e^{x^T \beta^{(i)}}}, \quad k = 1, \dots, K-1, \tag{7.9}$$

$$p_0(x) = \frac{1}{-\,||\,-}.$$

↳ ako su procjene $\hat{\beta}^{(1)}, \dots, \hat{\beta}^{(K-1)}$ te $\hat{\beta}^{(0)} := (0, \dots, 0) \in \mathbb{R}^p$,

$$\boxed{\hat{p}_k(x) := \frac{e^{x^T \hat{\beta}^{(k)}}}{\sum_{i=0}^{K-1} e^{x^T \hat{\beta}^{(i)}}}}, \quad k = 0, 1, \dots, K-1.$$

[tj. 0-1 gubitak]

↳ ako je $\hat{f}(x) = \underset{k \in S}{\arg\max}\, \hat{p}_k(x)$, imamo

$$\hat{f}(x) = \underset{k \in S}{\arg\max}\, \underbrace{x^T \hat{\beta}^{(k)}}_{=: \hat{\delta}_k(x)}, \quad x \in \mathbb{R}^p$$

"diskriminacijske f-je"

↳ Granica odluke između klasa $k, l \in S$ je

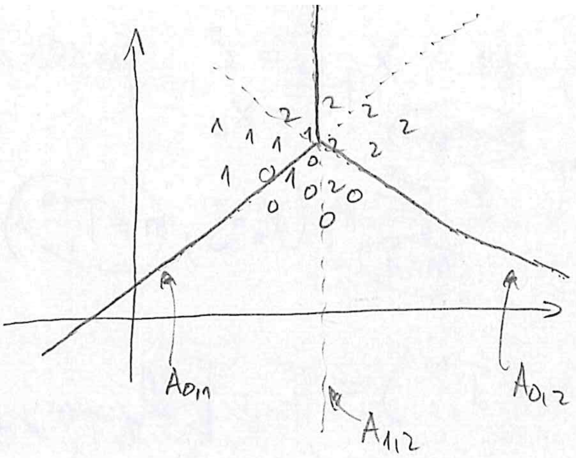
$$A_{k,l} = \{x \in \mathbb{R}^p : \hat{\delta}_k(x) = \hat{\delta}_l(x)\} = \{x : x^T(\hat{\beta}^{(k)} - \hat{\beta}^{(l)}) = 0\}$$

hiperravnina u $\mathbb{R}^p$, $\forall k, l$

↳ linearne granice odluke!

[tipično imamo i $x_0 = 1$ pa $A_{k,l}$ ne prolazi nužno kroz 0.]

npr.)




$K=3, \ p=2$

$1^\circ$ $A_{0,1}$ i $A_{0,2}$

$\rightarrow$ nacrtaj $\{x : f(x)=0\}$

$2^\circ$ $A_{1,2}$ $\rightarrow$ $\{f(x)=1\}$ i $\{f(x)=2\}$

[Moraju li se $A_{ij}$ sjeći u jednoj točci? DA]