

(6) Neprekidne slučajne varijable

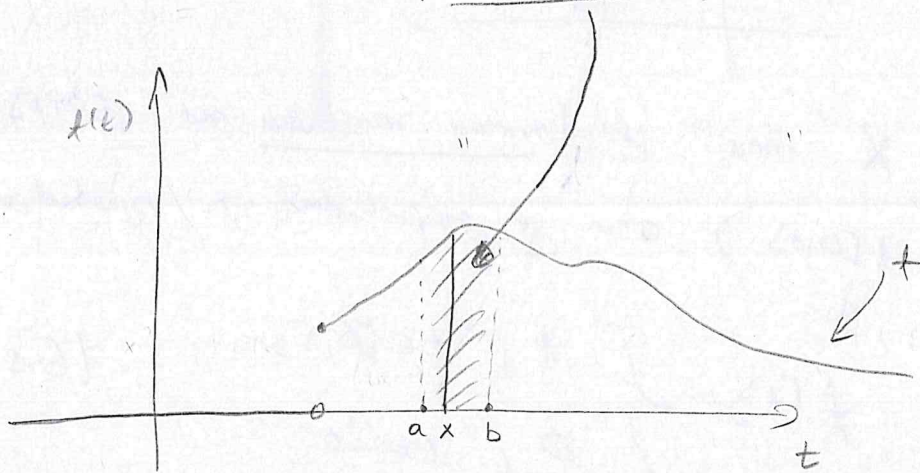
Def. 6.1) Sluč. var. $X: \Omega \rightarrow \mathbb{R}$ je (apsolutno) neprekidna ako postoji f-ja $f: \mathbb{R} \rightarrow [0, +\infty)$ t.d.

$$\boxed{P(X \leq x) = \int_{-\infty}^x f(t) dt, \quad \forall x \in \mathbb{R}.} \quad (6.1)$$

U tom slučaju, f zovemo funkcija gustobe od X.

Iz (6.1) sledi da za sve $a, b \in \mathbb{R}, a < b$, vrijedi:

$$\begin{aligned} \boxed{P(a < X \leq b)} &= P(\{X \leq b\} \setminus \{X \leq a\}) \\ &= P(X \leq b) - P(X \leq a) \\ &\stackrel{(6.1)}{=} \boxed{\int_a^b f(t) dt,} \end{aligned} \quad (6.2)$$



Općenito, za "određenu" $B \subseteq \mathbb{R}$, vrijedi:

$$P(X \in B) = \int_B f(t) dt \quad (6.3)$$

impr.:

• za $x \in \mathbb{R}$,

$$\boxed{P(X=x) \stackrel{(6.3)}{=} \int_x^x f(t) dt = 0} \quad (6.4)$$

• za sve $a, b \in \mathbb{R}, a < b,$

$$P(X=a) = P(X=b) = 0 \quad (6.4)$$

$$P\left(a \stackrel{\leq}{\leq} X \stackrel{\leq}{\leq} b\right) \stackrel{\text{ili}}{=} P(a < X \leq b) = \int_a^b f(t) dt.$$

(6.3)

• $\forall a \in \mathbb{R},$

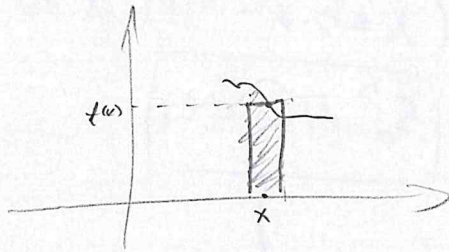
$$P(X \stackrel{\geq}{\geq} a) = \int_a^{\infty} f(t) dt.$$

[$f(t)$ nije negativna!]

Nap. (intuicija o f -ji gustoće)

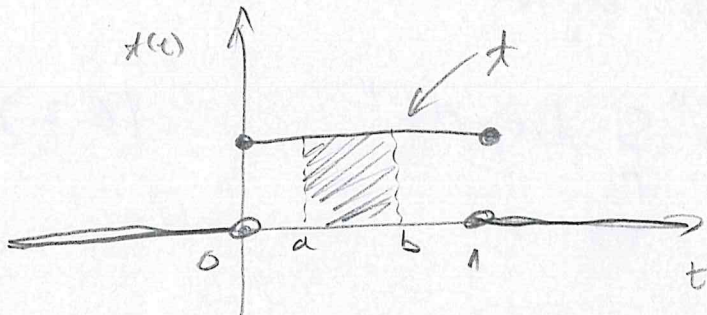
iz (6.3) imamo za $x \in \mathbb{R}$ "male" $\Delta x,$

$$P(X \in [x - \Delta x, x + \Delta x]) = \int_{x - \Delta x}^{x + \Delta x} f(t) dt \approx 2\Delta x \cdot \boxed{f(x)}.$$



Pr. 6.2 | Sl. var. X ima uniformnu raspodelu na $[0, 1]$
(oznaka $X \sim \text{Unit}[0, 1]$) ako je neprekidna i gustoćom

$$f(t) = \begin{cases} 1, & t \in [0, 1] \\ 0, & \text{inače.} \end{cases} \quad (6.5)$$



Uočimo,

$$P(X \notin [0, 1]) = \int_{\text{inače}} f(t) dt = 0,$$

(0, $\forall t \in [0, 1]$)

te za $0, b \in [0, 1], a \leq b,$

$$P(X \in [a, b]) = \int_a^b \underbrace{f(t)}_{=1, \forall t \in [a, b]} dt = b - a = \frac{\lambda([a, b])}{\lambda(\mathbb{R})}$$

$\Rightarrow X$ je "slučajno" odabran broj iz \mathbb{R} ! ↑
geometrijska
uniformnost!

Uočimo, svaka funkcija gustoće $f: \mathbb{R} \rightarrow [0, \infty)$ zadovoljava

$$\int_{-\infty}^{\infty} f(t) dt \stackrel{(6.3)}{=} P(X \in \mathbb{R}) = 1.$$

Def. 1 Vrijedi: znat, svaka takva f -ja je f -ju gustoće
neke neprekidne slučajne varijable. ▣

Def. 1 F -ju gustoće odreduje "distribuciju" neprekidne
slučajne varijable. ▣

