JORDAN PRODUCT PRESERVING MAPS ON SIMPLE JORDAN ALGEBRAS

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Abstract. Every simple finite-dimensional formally real Jordan algebra is isomorphic to one of the following: algebra of $n \times n$ hermitian matrices over the real numbers $\mathbb{R}$, complex numbers $\mathbb{C}$ or quaternions $\mathbb{H}$, algebra of 3×3 hermitian matrices over the octonions $\mathbb{O}$, or algebra of spin factors Spin_n with $n \geq 2$.

In this talk we describe maps on this type of algebras with the property $\phi(A \circ B) = \phi(A) \circ \phi(B)$ for all elements A and B, i.e., maps that preserve the Jordan product. Although we do not assume additivity or bijectivity, it turns out that any such map is either a real linear automorphism or a constant map. On the way to our main result we also discuss several auxiliary results, including a description of zero Jordan product preserving maps, a generalization of Uhlhorn's theorem to octonions, and some results on ortographs of simple Jordan algebras. In the octonionic case we briefly mention the fundamental theorem of octonionic projective geometry and its connections to our results.

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