Zad 6, poglavlje 3.4.

a) Pretpostavimo da postoje $x, y \in \mathbb{R}$

t.d. $A = [I]_{(a,e)}$

$$\Rightarrow \quad A^{-1} = [I]_{(e,a)} = \begin{bmatrix} 0 & 1 & 0 \\ x & 0 & 1 \\ 0 & 0 & y \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 0 & 1 & -1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ x & 0 & 1 \\ 0 & 0 & y \end{bmatrix}$$

$$\Rightarrow I = \underbrace{\begin{bmatrix} 0 & 1 & -1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{A} \cdot \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ x & 0 & 1 \\ 0 & 0 & y \end{bmatrix}}_{A^{-1}} = \begin{bmatrix} x & 0 & 1-y \\ 0 & 1 & 0 \\ 0 & 0 & y \end{bmatrix}$$

$\Rightarrow \quad x = y = 1$ ✓ Postoji, to

je baza $(a) = \{(0,1,0), (1,0,0), (0,1,1)\}$

b) Pretpostavimo da postoje. Tada je

$$B = [I]_{(a,e)}$$

$$B^{-1} = [I]_{(e,a)}$$

$$I = BB^{-1} = \begin{bmatrix} 0 & -1 & 2 \\ 1 & 2 & -3 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ x & 0 & 1 \\ 0 & 0 & y \end{bmatrix}$$

$$= \begin{bmatrix} -x & 0 & -1+2y \\ 2x & 1 & 2-3y \\ 0 & 1 & y+1 \end{bmatrix}$$

Ova matrica $\neq I \ \forall x, y$

Ne postoje $x, y \in \mathbb{R}$ t.d. $B = [I]_{(a,e)}$

7. Zad, poglavlje 3.4.

a) $T(ax^2+bx+c)=(a+2b+3c)x + (a+b+2c)$

$(e)=\{1, x, x^2\}, \quad (f)=\{1, x\}$

$T(1) = 3x+2 = 2\cdot 1+3\cdot x$

$T(x) = 2x+1 = 1\cdot 1+2\cdot x$

$T(x^2) = x+1 = 1\cdot 1+1\cdot x$

$$[T]_{(f,e)} = \begin{array}{c} \begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 1 \end{bmatrix} \\ T(1) \quad T(x) \quad T(x^2) \end{array}$$

$(p)=\{p_1, p_2, p_3\}, \quad (q)=\{q_1, q_2\}$

$$[T]_{(q,p)} = [I]_{(q,f)}\cdot[T]_{(f,e)}\cdot[I]_{(e,p)}$$

$$= [I]_{(f,q)}^{-1}\cdot[T]_{(f,e)}\cdot[I]_{(e,p)}$$

$$\Rightarrow [T]_{(g,p)} = \begin{bmatrix} 2 & 1 \\ -1 & -1 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 1 \end{bmatrix}$$

$$\cdot \begin{bmatrix} 1 & -1 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -2 & -8 \\ -8 & 3 & 13 \end{bmatrix}$$

b) $[T(p)]_{(g)} = [T]_{(g,e)} \cdot [p]_{(e)}$

$$= [I]_{(g,f)} \cdot [T]_{(f,e)} \cdot [p]_{(e)}$$

$$= \begin{bmatrix} 2 & 1 \\ -1 & -1 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

c) Želimo naći baze (e') za $\mathcal{P}_2$

i (f') za $\mathcal{P}_1$ t.d. vrijedi

$$[T]_{(f',e')} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix},$$

$r(T) = r([T]_{(f,e)})$

$= 2$

tj. $T(e_1') = f_1'$

$T(e_2') = f_2'$

$T(e_3') = 0$

$e_3' \in \operatorname{Ker} T \quad T(ax^2+bx+c) = 0 \iff$

$a+2b+3c=0, \; a+b+2c=0$

$\iff \; a = b = -c$

Uzmemo $e_3'(x) = -1 + x + x^2 \quad T(e_3') = 0$

Za $\{e_1', e_2'\}$ uzmemo bilo koje t.d.

$\{e_1', e_2', e_3'\}$ bude baza za $\mathcal{P}_2$ i da

$\{f_1', f_2'\} = \{T(e_1'), T(e_2')\}$ bude baza za $\mathcal{P}_1$,

npr. $e_1'(x) = 1$, $e_2'(x) = x$

$$f_1'(x) = T(e_1') = 2+3x$$

$$f_2'(x) = T(e_2') = 1+2x$$

$$[T]_{(f', e')} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$T(e_1') = f_1' = 1 \cdot f_1' + 0 \cdot f_2'$ $\quad$ $T(e_2') = f_2' = 0 \cdot f_1' + 1 \cdot f_2'$ $\quad$ $T(e_3') = 0$