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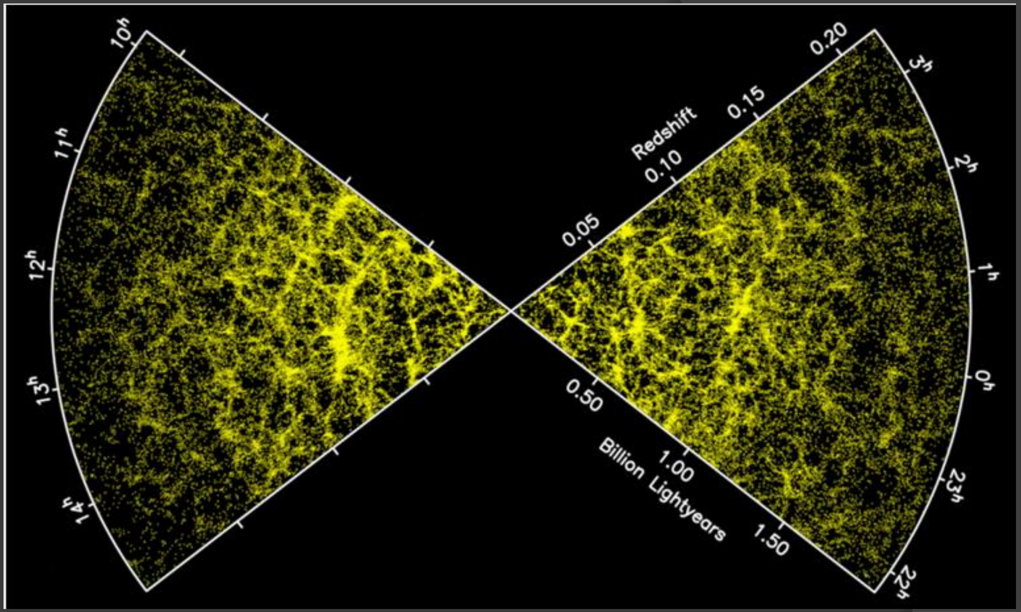
NOVI FORMALIZAM ZA STATISTIČKA SVOJSTVA KOZMIČKE STRUKTURE VELIKIH SKALA

Uvod

- Veliki prasak, inflacija Svemira
- Epoha gotovo glatkog Svemira (anizotropije *CMB*-a)
- Kvantne fluktuacije prostora tijekom inflacije
- Evolucija nehomogenosti u strukturu
- Teorijski opis procesa evolucije
- ΛCDM

Uvod



https://www.roe.ac.uk
od The Royal
Observatory, Edinburgh

- *2dF* popis
- 4 mlrd. ly
  1226 Mpc
- Velike skale (>10 Mpc)
- Malo odstupanje od glatkog Svemira
- Račun smetnje
- Male skale, nelinearnosti

Matematički objekti i notacija

$$\int_{\mathbf{x}\in\mathbb{R}^3} f(\tau,\mathbf{x})g(\tau,\mathbf{x},\mathbf{y}) \equiv f(\tau,\mathbf{x})g(\tau,\mathbf{x},\mathbf{y})$$

$$\begin{aligned} A^I_{\mu\alpha}(\tau,\mathbf{x},\mathbf{y})A_{\alpha\nu}(\tau,\mathbf{y},\mathbf{z}) &= A_{\nu\alpha}(\tau,\mathbf{z},\mathbf{y})A^I_{\alpha\mu}(\tau,\mathbf{y},\mathbf{x}) \\ &= \delta_{\mu\nu}\delta_D(\mathbf{x}-\mathbf{z}) \end{aligned}$$

(**funkcionalni inverz polja matrica**)

$$A^I_{\mu\nu}(\tau,\mathbf{x},\mathbf{y}) = A^I_{\nu\mu}(\tau,\mathbf{y},\mathbf{x})$$

Matematički objekti i notacija

$$\int_{[w]} f(w)\mathbb{G}\{w|A_{\mu\nu}\} \qquad \mathbb{G}\{w|A_{\mu\nu}\} = \exp\left[-\frac{1}{2} w_\mu(\tau,\mathbf{x}) A_{\mu\nu}(\tau,\mathbf{x},\mathbf{y}) w_\nu(\tau,\mathbf{y})\right]$$

$$\frac{\int_{[w]} f(w)\mathbb{G}\{w|A_{\mu\nu}\}}{\int_{[w]} g(w)\mathbb{G}\{w|A_{\alpha\beta}\}} \equiv \lim_{n\to\infty} \frac{\int_{w(\tau,\mathbf{x}_1)} \cdots \int_{w(\tau,\mathbf{x}_n)} f_n(w)\exp[-\frac{1}{2}\lambda_n^2 \sum_{j,l=1}^n w_\mu(\tau,\mathbf{x}_j) A_{\mu\nu}(\tau,\mathbf{x}_j,\mathbf{x}_l) w_\nu(\tau,\mathbf{x}_l)]}{\int_{w(\tau,\mathbf{x}_1)} \cdots \int_{w(\tau,\mathbf{x}_n)} g_n(w)\exp[-\frac{1}{2}\lambda_n^2 \sum_{j,l=1}^n w_\alpha(\tau,\mathbf{x}_j) A_{\alpha\beta}(\tau,\mathbf{x}_j,\mathbf{x}_l) w_\beta(\tau,\mathbf{x}_l)]}$$

(**funkcionalni omjer**)

$$w_\mu(\tau,\mathbf{x}) = w'_\mu(\tau,\mathbf{x}) + iA^I_{\mu\nu}(\tau,\mathbf{x},\mathbf{y})J_\nu(\mathbf{y})$$

(**eliminirajuća supstitucija**)

Matematički objekti i notacija

$$\int_{[X_{\mu_1}]} \cdots \int_{[X_{\mu_m}]} X_{\mu_1}(\tau, \mathbf{x}_1) \ldots X_{\mu_m}(\tau, \mathbf{x}_m) \mathbb{P}_X$$

$$\equiv \langle X_{\mu_1}(\tau, \mathbf{x}_1) \ldots X_{\mu_m}(\tau, \mathbf{x}_m) \rangle$$

(**m−korelator od** $X_{\mu_1}, \ldots, X_{\mu_m}$)

Polazne jednadžbe

- Opća teorija relativnosti + kinetička teorija, deterministički

$$\rho(\tau, \mathbf{x}) = \overline{\rho}(\tau)(1 + \delta(\tau, \mathbf{x})) \qquad \text{(kontrast gustoće)}$$

$$\partial_\tau \delta(\tau, \mathbf{x}) + \nabla \cdot [\mathbf{v}(1 + \delta)](\tau, \mathbf{x}) = 0,$$

$$\partial_\tau \mathbf{v}(\tau, \mathbf{x}) + (\mathbf{v}\nabla)\mathbf{v}(\tau, \mathbf{x}) + \mathcal{H}(\tau)\mathbf{v}(\tau, \mathbf{x}) = -\nabla\Phi(\tau, \mathbf{x}),$$

$$\nabla^2 \Phi(\tau, \mathbf{x}) = 4\pi G a^2(\tau) \overline{\rho}(\tau) \delta(\tau, \mathbf{x})$$

$$\begin{cases} \partial_\tau \tilde{\delta}(\tau, \mathbf{k}) + \tilde{\theta}(\tau, \mathbf{k}) \\ \qquad + E(\mathbf{k}, \mathbf{q}, \mathbf{u}) \tilde{\delta}(\tau, \mathbf{q}) \tilde{\theta}(\tau, \mathbf{u}) = 0 \\ D\tilde{\delta}(\tau, \mathbf{k}) + \partial_\tau \tilde{\theta}(\tau, \mathbf{k}) + \mathcal{H}(\tau) \tilde{\theta}(\tau, \mathbf{k}) \\ \qquad + F(\mathbf{k}, \mathbf{q}, \mathbf{u}) \tilde{\theta}(\tau, \mathbf{q}) \tilde{\theta}(\tau, \mathbf{u}) = 0 \end{cases}$$

Polazne jednadžbe

- Polispektri opisuju statistička svojstva koja se mogu mjeriti
- Spektar snage, f. t. 2-korelatora

$$\int_{\mathbf{x}-\mathbf{y}} \langle X_\mu(\tau,\mathbf{x}) X_\nu(\tau,\mathbf{y})\rangle \, e^{-i\mathbf{k}(\mathbf{x}-\mathbf{y})} \equiv P_{X_\mu X_\nu}(\tau,k)$$

$$\langle \tilde{X}_\mu(\tau,\mathbf{k}) \tilde{X}_\nu(\tau,\mathbf{q})\rangle = (2\pi)^3 \delta_D(\mathbf{k}+\mathbf{q}) P_{X_\mu X_\nu}(\tau,k)$$

- Potreban je **formalizam** za povezati statistička svojstva s determinističkim jednadžbama

Pregled starog formalizma

- Kvantne fluktuacije prostora tijekom inflacije
- Gustoća vjerojatnosti fluktuacija približno Gausijanska
- Kontrast nasljeđuje svojstva

$$\mathbb{P}_{\delta_0} = \frac{1}{\mathcal{N}\{1|\delta_0|U\}}\mathbb{G}\{\delta_0|U\}$$

- Ostale veličine nasljeđuju od kontrasta

Pregled starog formalizma

$$\zeta(J) = \exp[\frac{-(2\pi)^3}{2}\int_{\mathbf{k}} J(\mathbf{k})P_{\delta_0}(k)J(-\mathbf{k})]$$

$$\langle\delta_0(\mathbf{x}_1)...\delta_0(\mathbf{x}_m)\rangle = \frac{1}{i^m}\Delta^m_{J(\mathbf{x}_1)...J(\mathbf{x}_m)}Z(0)$$

$$X(\tau,\mathbf{k}) = \sum_{n=1}^{\infty}\int_{\mathbf{k}_1}...\int_{\mathbf{k}_n}\frac{D_+^n(\tau)\delta_D(\mathbf{k}-\sum_{j=1}^n \mathbf{k}_j)}{(2\pi)^{3(n-1)}} \times I_n(\mathbf{k}_1,...,\mathbf{k}_n)\tilde{\delta}_0(\mathbf{k}_1)...\tilde{\delta}_0(\mathbf{k}_n)$$

$$\langle X(\tau,\mathbf{k})X(\tau,\mathbf{q})\rangle \approx (2\pi)^3\delta_D(\mathbf{k}+\mathbf{q}) \times I_1(\mathbf{k})P_L(\tau,k)I_1(\mathbf{q})$$

Novi formalizam

$$L_{\mu\nu}\Phi_\nu(\tau,\mathbf{k}) + \gamma_{\mu\nu\eta}(\mathbf{k},\mathbf{q},\mathbf{u})\Phi_\nu(\tau,\mathbf{q})\Phi_\eta(\tau,\mathbf{u}) = 0$$

(**srednji dublet strukture**)

$$\begin{pmatrix} \partial_\tau\Phi_1(\tau,\mathbf{k}) + \Phi_2(\tau,\mathbf{k}) \\ \mathcal{D}\Phi_1(\tau,\mathbf{k}) + \partial_\tau\Phi_2(\tau,\mathbf{k}) + \mathcal{H}(\tau)\Phi_2(\tau,\mathbf{k}) \end{pmatrix} = \begin{pmatrix} -E(\mathbf{k},\mathbf{q},\mathbf{u})\Phi_1(\tau,\mathbf{q})\Phi_2(\tau,\mathbf{u}) \\ -F(\mathbf{k},\mathbf{q},\mathbf{u})\Phi_2(\tau,\mathbf{q})\Phi_2(\tau,\mathbf{u}) \end{pmatrix}$$

Novi formalizam

- Nepoznata veličina, slučajno polje, koja opisuje „izvor” strukture

$\epsilon_\mu(\tau, \mathbf{k})$ (ϵ-**fenomen**)

$$\mathbb{P}_{\tilde{\epsilon}} = \frac{1}{\mathcal{N}\{1|\tilde{\epsilon}|N^I_{\alpha\beta}\}} \mathbb{G}\{\tilde{\epsilon}|N^I_{\mu\nu}\}$$

- Stalna Gausijanska gustoća vjerojatnosti
- Prostorni „šum” koji generira kozmičku strukturu

Novi formalizam

$$L_{\mu\nu}\phi_\nu(\tau,\mathbf{k}) + \gamma_{\mu\nu\eta}(\mathbf{k},\mathbf{q},\mathbf{u})\phi_\nu(\tau,\mathbf{q})\phi_\eta(\tau,\mathbf{u}) = \epsilon_\mu(\tau,\mathbf{k})$$

$$L_{\mu\nu}\int_{[\epsilon]}\phi_\nu[\epsilon(\tau,\mathbf{k})]\mathbb{P}_\epsilon + \int_{[\epsilon]}\gamma_{\mu\nu\eta}(\mathbf{k},\mathbf{q},\mathbf{u})\phi_\nu[\epsilon(\tau,\mathbf{q})]\phi_\eta[\epsilon(\tau,\mathbf{u})]\mathbb{P}_\epsilon = \langle\epsilon_\mu(\tau,\mathbf{k})\rangle$$

$$\Phi(\tau,\mathbf{k}) = \int_{[\epsilon]}\phi[\epsilon(\tau,\mathbf{k})]\mathbb{P}_\epsilon$$

(dublet strukture)

$$\gamma_{\mu\nu\eta}(\mathbf{k},\mathbf{q},\mathbf{u})\left\langle\phi_\nu[\epsilon(\tau,\mathbf{q})]\phi_\eta[\epsilon(\tau,\mathbf{u})]\right\rangle_\epsilon$$
$$= \gamma_{\mu\nu\eta}(\mathbf{k},\mathbf{q},\mathbf{u})\left\langle\phi_\nu[\epsilon(\tau,\mathbf{q})]\right\rangle_\epsilon\left\langle\phi_\eta[\epsilon(\tau,\mathbf{u})]\right\rangle_\epsilon$$

???

Novi formalizam

- Račun smetnje na izvodnici

$$Z(J,M)=\frac{\int_{[\phi]}\exp[iJ_\mu(\mathbf{k})\phi_\mu(\tau,\mathbf{k})]\int_{[\chi]}\exp[iM_\mu(\mathbf{k})\chi_\mu(\tau,\mathbf{k})]\exp[i\chi_\mu(\tau,\mathbf{k})L_{\mu\nu}\phi_\nu(\tau,\mathbf{k})+S(\tau)]\mathbb{G}\{\chi|K_{\eta\sigma}\}}{\int_{[\phi]}\int_{[\chi]}\exp[i\chi_\omega(\tau,\mathbf{k})L_{\omega\xi}\phi_\xi(\tau,\mathbf{k})+S(\tau)]\mathbb{G}\{\chi|K_{\alpha\beta}\}}$$

$$\exp[S(\tau)]=\sum_{j=0}^{\infty}\frac{1}{j!}S^j(\tau)$$

- Najniži red

$$Z_0(J,M)=\exp\{-iJ_\mu(\mathbf{k})B^{-1}_{\mu\nu}(\tau)[D_{\nu\eta}\phi_\eta(\tau,\mathbf{k})+M_\nu(\mathbf{k})]\}\mathbb{G}\{J|C^I_{\alpha\beta}\}$$

Novi formalizam

$$\langle \phi_\mu(\tau,\mathbf{k})\phi_\nu(\tau,\mathbf{q})\rangle \approx (2\pi)^3\delta_D(\mathbf{k}+\mathbf{q}) \times B^{-1}_{\mu\alpha}(\tau)P_{\tilde{\epsilon}_\alpha\tilde{\epsilon}_\beta}(\tau,k)B^{-T}_{\beta\nu}(\tau)$$

$$\langle X(\tau,\mathbf{k})X(\tau,\mathbf{q})\rangle \approx (2\pi)^3\delta_D(\mathbf{k}+\mathbf{q}) \times I_1(\mathbf{k})P_L(\tau,k)I_1(\mathbf{q})$$

$$P_L(\tau,k) = B^{-1}_{1\mu}(\tau)P_{\tilde{\epsilon}_\mu\tilde{\epsilon}_\nu}(\tau,k)B^{-T}_{\nu 1}(\tau)$$