# Proučavanje ravnoteže sila i momenata u diobenom vretenu u aproksimaciji srednjeg polja

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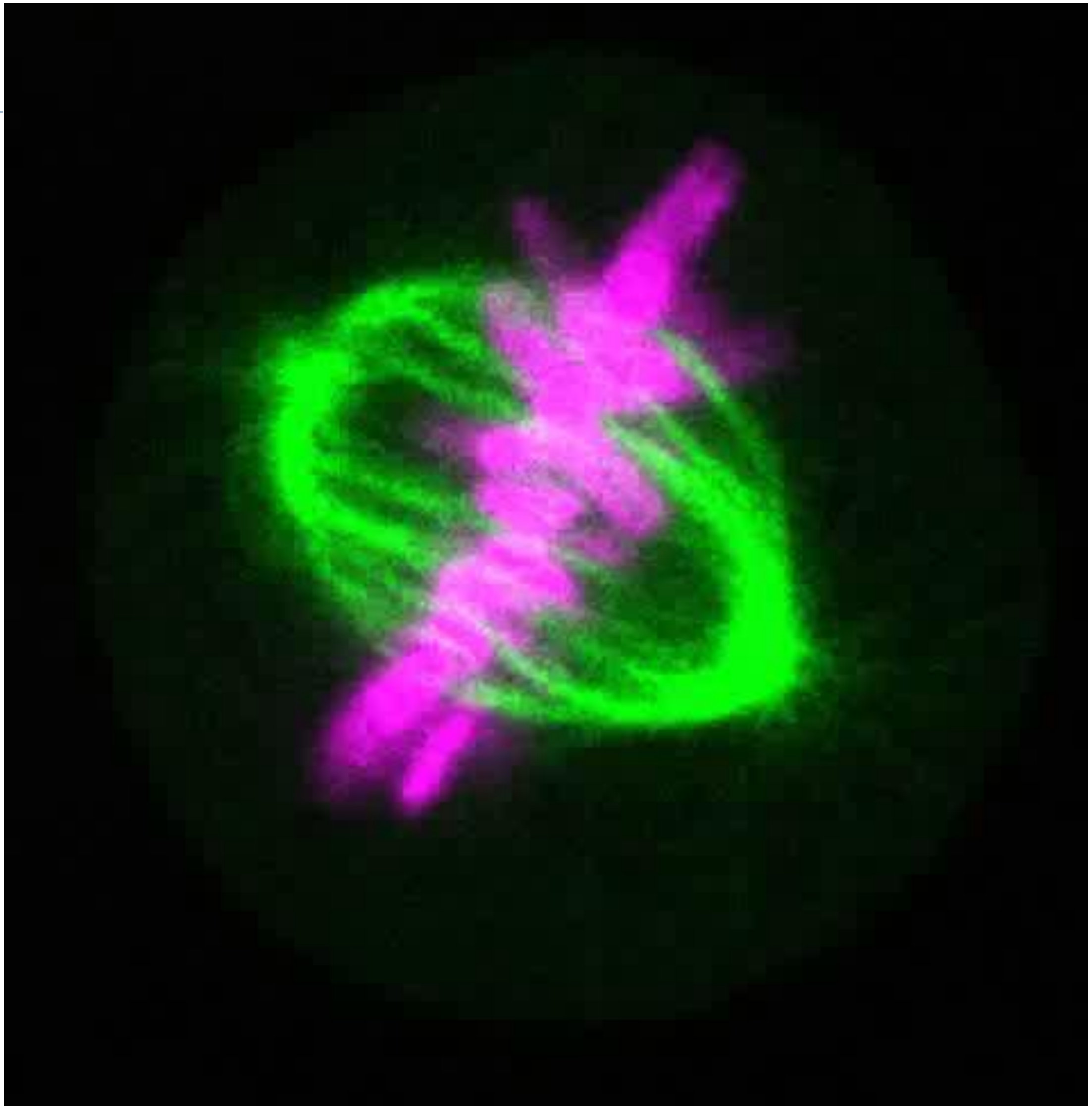
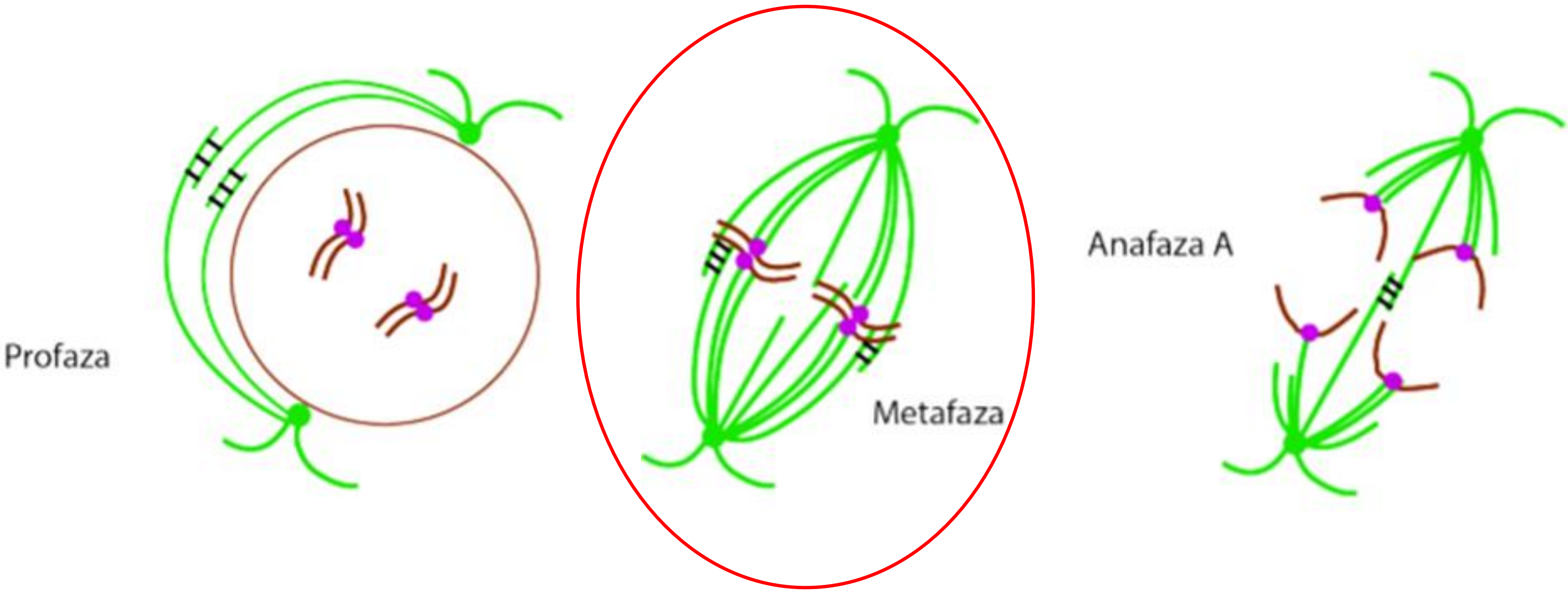
# Stanična dioba

- Dioba tjelesnih stanica - mitoza
- Stanica majka se djeli na dvije stanice kćeri



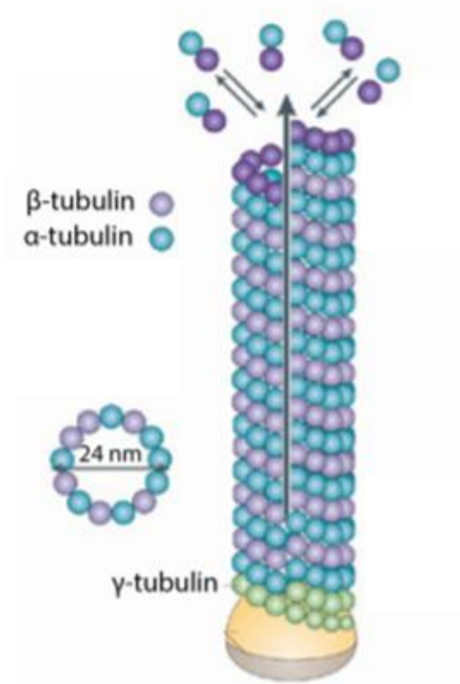
# Faze diobe




Alberts, B., Johnson, A.

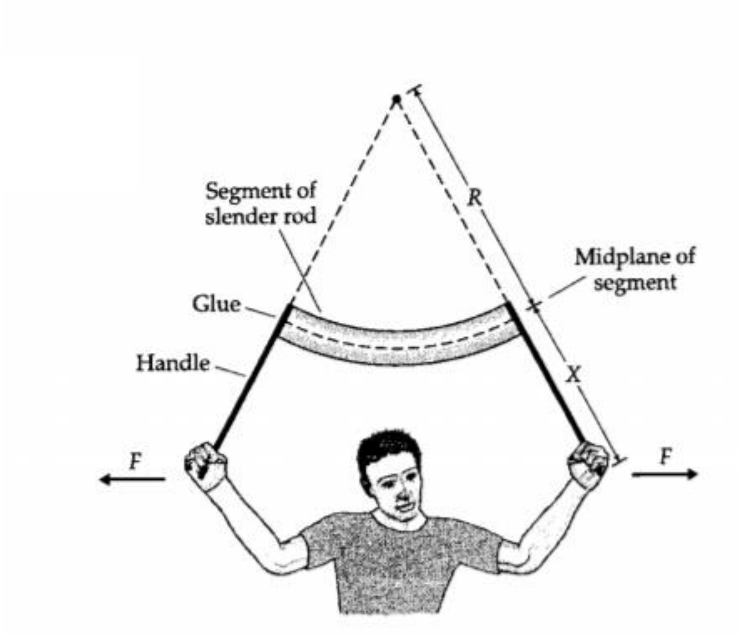
Molecular Biology of the Cell

# Svojstva mikrotubula



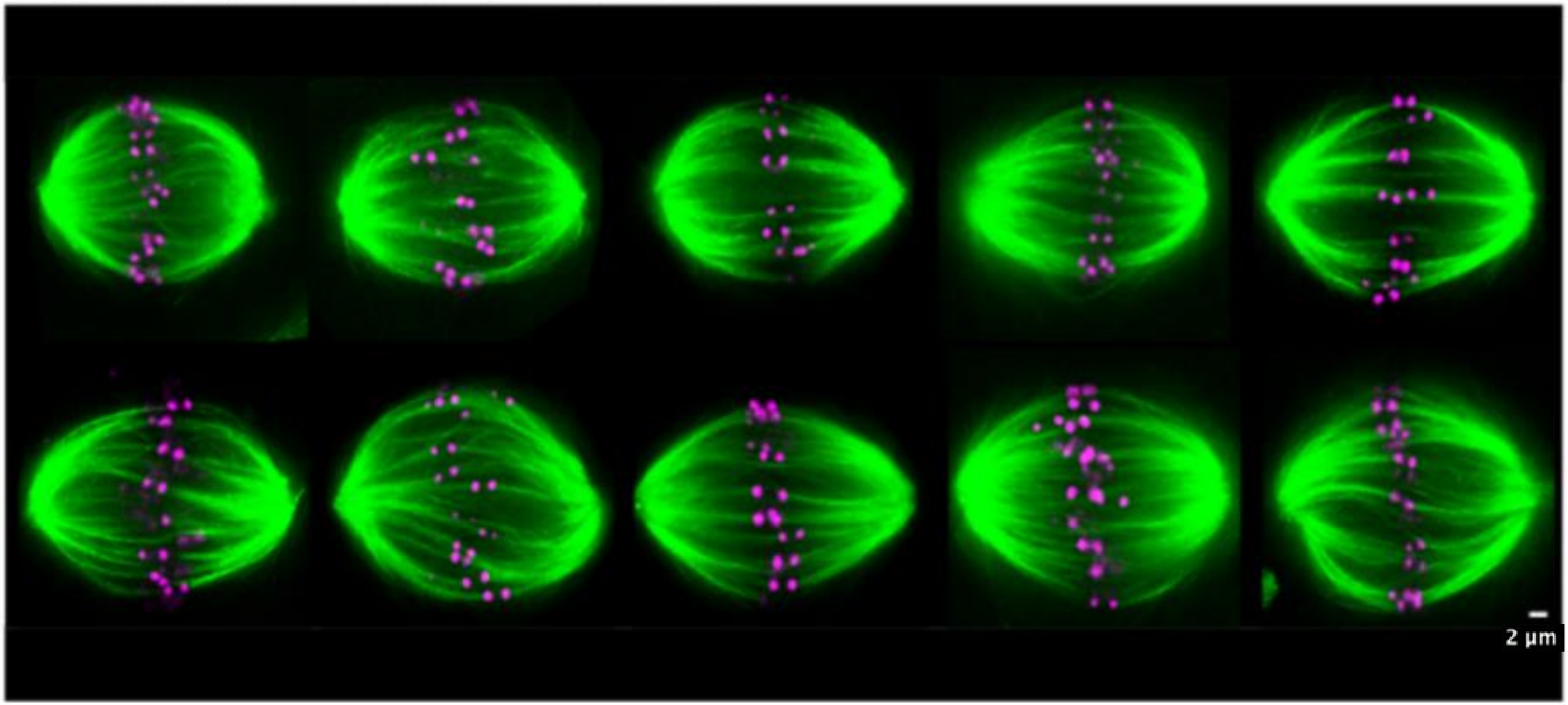

Rochlin, M. W., Dailey, M. E. & Bridgman, P. C. Polymerizing microtubules activate site-directed F-actin assembly in nerve growth cones. *Mol. Biol. Cell* **10**, 2309-2327 (1999)

- Elastičan štap




Howard J.,
Mechanics of Motor Proteins & the Cytoskeleton

# Izgled stanice u metafazi

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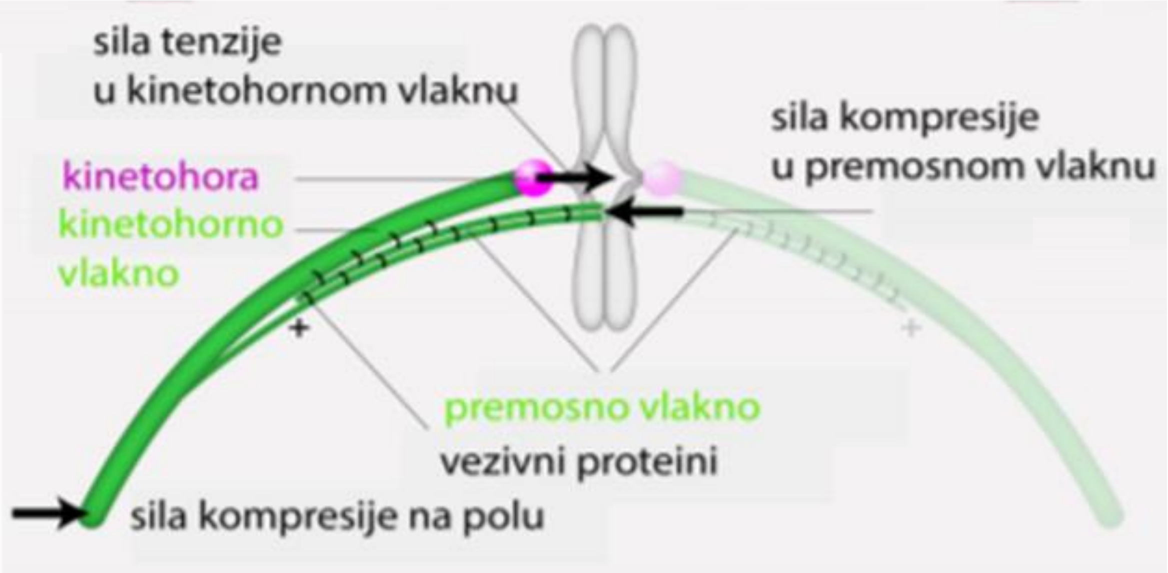
# Premosni mikrotubuli

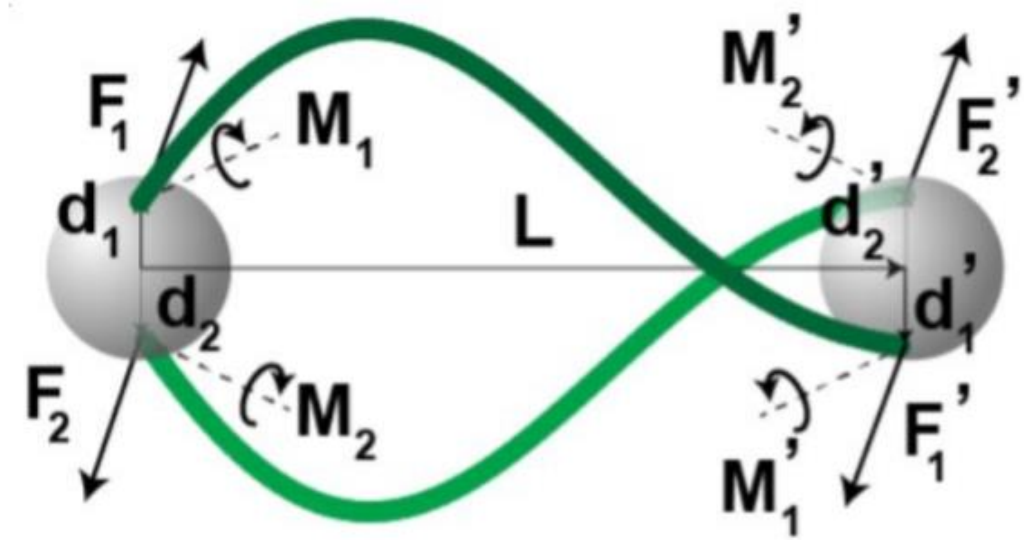
- Dio mikrotubula se spaja i formira premosno vlakno




- Tretiramo kao jedan jedinstveni objekt

Tolić, I. M. and Pavin, N. Bridging the gap between sister kinetochores *Cell Cycle* **15** 1169-1170 (2016)

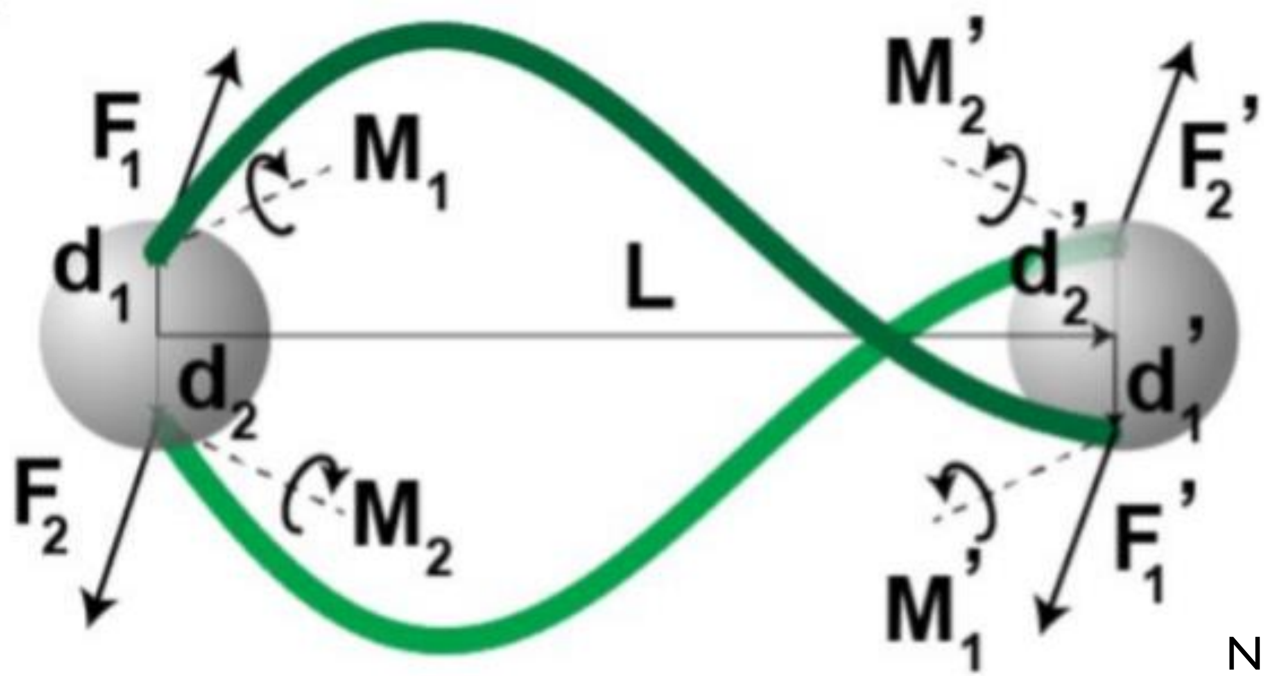
# Diskretni model ravoteže




- Novak *et al.* : statička ravnoteža u metafazi

- Izjednačene sile i momenti na centrosomu i na mikrotubulu

# Jednadžbe ravnoteže



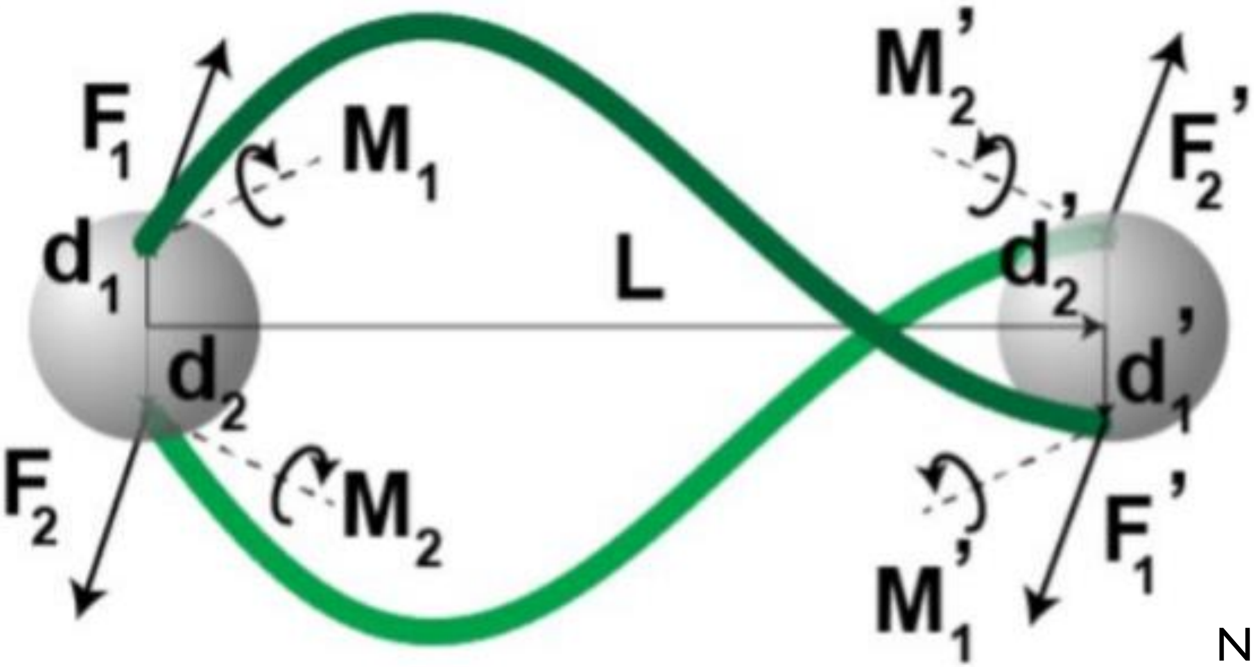

Novak et al.

Lijevi centrosom

$$\sum_{i}^{n} \vec{F}_i = 0,$$

$$\sum_{i}^{n} (\vec{M}_i + \vec{d}_i \times \vec{F}_i) = 0.$$

# Jednadžbe ravnoteže




Novak et al.

Lijevi centrosom

$$\sum_{i}^{n} \vec{F_i} = 0,$$

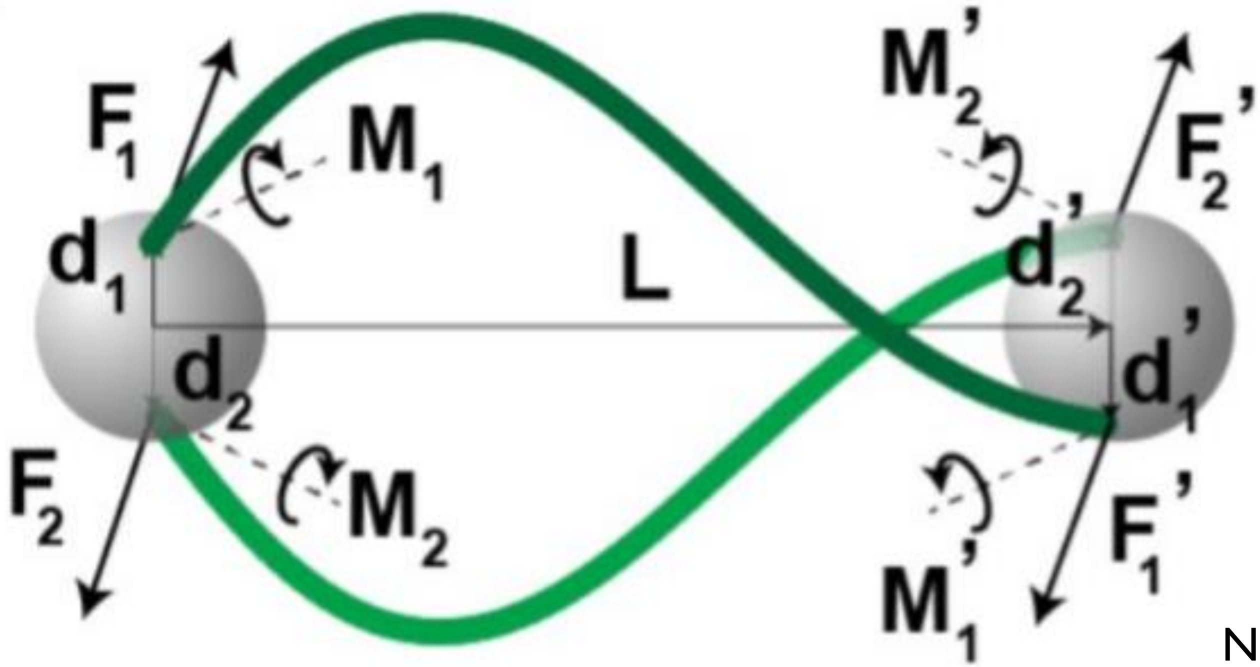
$$\sum_{i}^{n} (\vec{M_i} + \vec{d_i} \times \vec{F_i}) = 0.$$

Desni centrosom

$$\sum_{i}^{n} \vec{F_i'} = 0,$$

$$\sum_{i}^{n} (\vec{M_i'} + \vec{d_i'} \times \vec{F_i'}) = 0.$$

# Jednadžbe ravnoteže



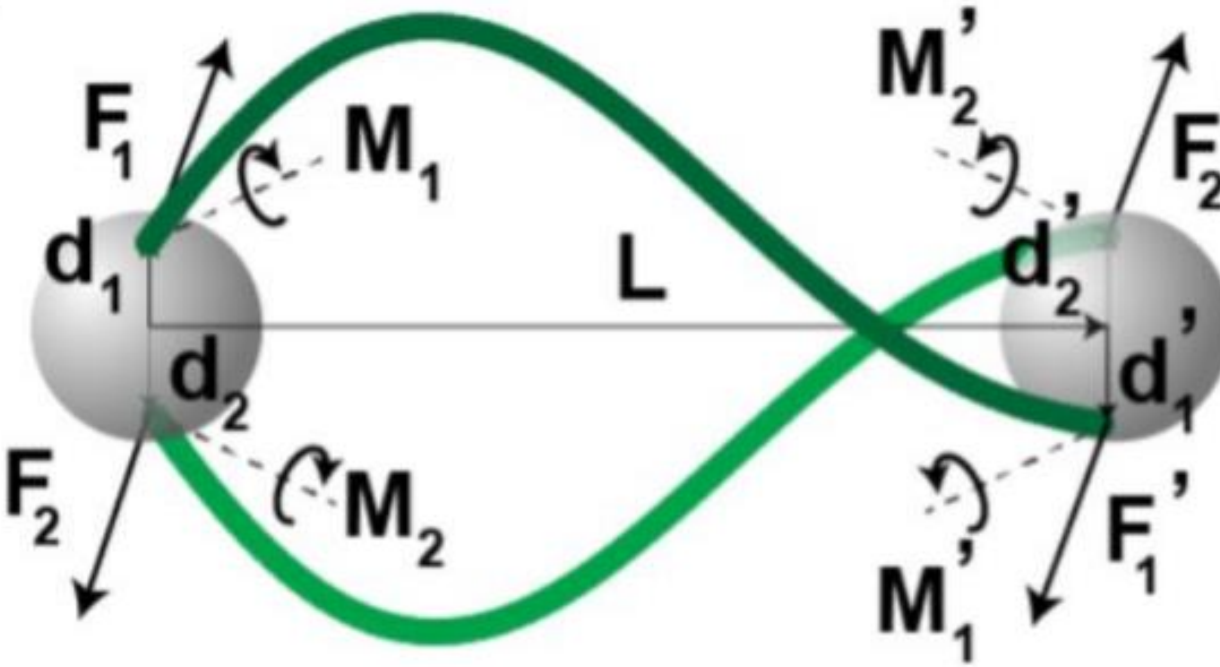

Novak et al.

Ravnoteža jednog mikrotubula

$$\vec{F}_i + \vec{F}_i' = 0,$$

$$\vec{M}_i + \vec{M}_i' + \vec{d}_i \times \vec{F}_i + (\vec{d}_i' + \vec{L}) \times \vec{F}_i' = 0.$$

# Jednadžbe ravnoteže




Novak et al.

Nije pogodno za opis čitavog vretena → kontinuirani model

# Kontinuirani model

- Prelazak na kontinurane varijable

$i = 1, \ldots, n$ → $\rho(\theta, \phi)$

$\vec{F}_i$ → $\vec{F}(\rho(\theta, \phi))$

$\vec{M}_i$ → $\vec{M}(\rho(\theta, \phi))$

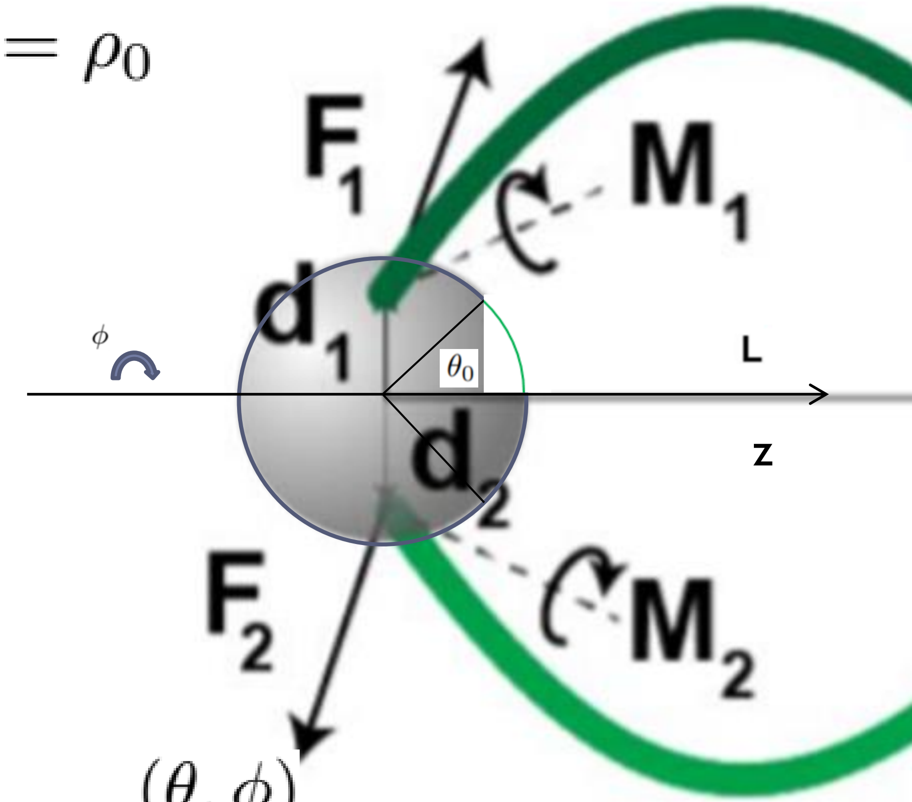
# Gustoća mikrotubula

- Homogeno raspodjeljeni po površeni sfere

$$\rho(\theta, \phi) = \rho_0$$

- Granični kut populiranosti

$$\theta_0$$




$$\rho(\theta, \phi) \Rightarrow (\theta, \phi)$$

# Simetrije

- Rotaciona simetrija oko osi spojnice

$$\vec{F}^{\perp}(\theta,\phi) = -\vec{F}^{\perp}(\theta,\phi+\pi),$$
$$\vec{F}^{\parallel}(\theta,\phi) = \vec{F}^{\parallel}(\theta,\phi+\pi),$$
$$\left|\vec{F}(\theta,\phi)\right| = \left|\vec{F}(\theta)\right|,$$

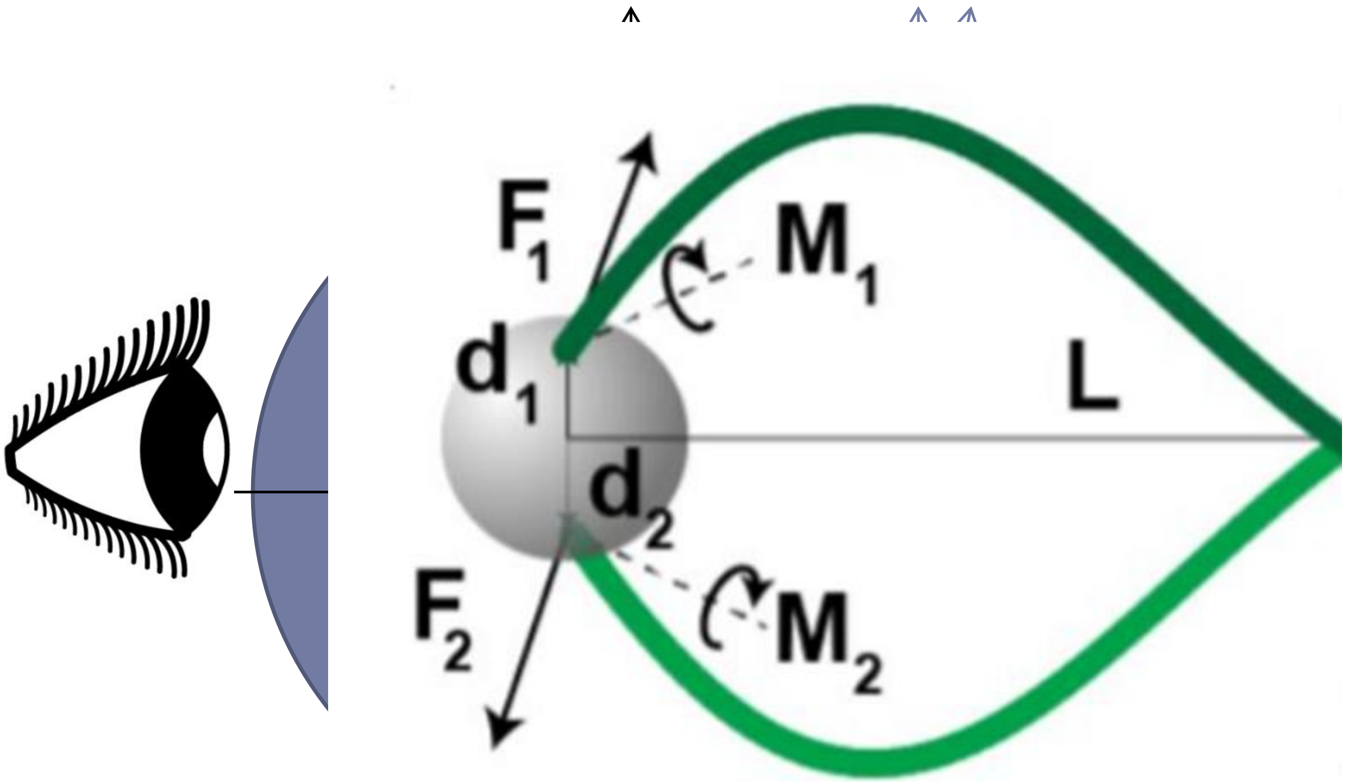
- Invarijantnost na zamjenu lijevog i desnog centrosoma

$$\theta' = +\theta$$

# Implementiranje simetrija

$$\vec{F}(\theta,\phi) = f^{\|}(\theta)\hat{z} + f^{\perp}(\theta)(cos(\phi+\delta)\hat{x} + sin(\phi+\delta)\hat{y}),$$

$$\vec{M}(\theta,\phi) = m^{\|}(\theta)\hat{z} + m^{\perp}(\theta)(cos(\phi+\delta)\hat{x} + sin(\phi+\delta)\hat{y}),$$



# Jednadžbe ravnoteže - centrosom

$$\sum_{i}^{n} \vec{F}_i = 0,$$

$$\int_0^{2\pi} \int_0^{\theta_0} \vec{F}(\theta, \phi) d\theta d\phi,$$

$$\hat{z} : \int_0^{2\pi} \int_0^{\theta^0} \vec{f}^{\|}(\theta) d\theta d\phi = \int_0^{\theta^0} \vec{f}^{\|}(\theta) d\theta = 0.$$

$$\sum_{i}^{n} (\vec{M}_i + \vec{d}_i \times \vec{F}_i) = 0.$$

$$\int_0^{2\pi} \int_0^{\theta_0} (\vec{M}(\theta, \phi) + \vec{d}(\theta, \phi) \times \vec{F}(\theta, \phi)) d\theta d\phi = 0.$$

$$\int_0^{\theta^0} \vec{m}^{\|}(\theta) d\theta = 0$$

# Jednadžbe ravnoteže – sile na mikrotubul

$$\vec{F}_i + \vec{F}_i' = 0,$$

$$\vec{F}(\theta, \phi) = -\vec{F}'(\theta', \phi'),$$

$$f^{\perp}(\theta) = \pm f^{\perp'}(\theta'),$$

$$\phi_1' = \phi + (\delta - \delta'),$$
$$\phi_2' = \phi + \pi + (\delta - \delta'),$$

- Veza položaja lijevog i desnog kraja MT-a

# Jednadžbe ravnoteže – momenti na mikrotubul

$$\vec{M}_i + \vec{M}_i' + \vec{d}_i \times \vec{F}_i + (\vec{d}_i' + \vec{L}) \times \vec{F}_i' = 0.$$

$$\vec{M}(\theta, \phi) + \vec{M}'(\theta', \phi') + \vec{d} \times \vec{F}(\theta, \phi) + (\vec{d}' + \vec{L}) \times \vec{F}'(\theta', \phi') = 0.$$

# Jednadžbe ravnoteže – momenti na mikrotubul

- Raspis po komponentama:

$$\hat{x} : m^{\perp}(\theta)cos(\phi+\delta) + m^{\perp'}(\theta')cos(\phi'+\delta') + d(sin(\theta)sin(\phi+\delta)f^{\parallel}$$
$$- cos(\theta)sin(\phi+\delta)f^{\perp}) + d'(sin(\theta')sin(\phi'+\delta')f^{\parallel'}$$
$$- cos(\theta')sin(\phi'+\delta')f^{\perp'}) - f^{\perp'}Lsin(\phi'+\delta') = 0,$$

$$\hat{y} : m^{\perp}(\theta)sin(\phi+\delta) + m^{\perp'}(\theta')sin(\phi'+\delta') + d(cos(\theta)sin(\phi+\delta)f^{\perp}$$
$$- sin(\theta)sin(\phi+\delta)f^{\parallel}) + d'(cos(\theta')sin(\phi'+\delta')f^{\perp'}$$
$$- sin(\theta')sin(\phi'+\delta')f^{\parallel'}) + f^{\perp'}Lcos(\phi'+\delta') = 0,$$

$$\hat{z} : m^{\parallel}(\theta) + m^{\parallel'}(\theta') = 0.$$

A
G

# Taylorov razvoj

- Rješavamo razvojem u Taylorov red

$$f^{\perp/\parallel} = f_0^{\perp} + f_1^{\perp}\theta + o(\theta^2)$$

$$m^{\perp/\parallel} = m_0^{\perp} + m_1^{\perp}\theta + o(\theta^2)$$

- Zadržavamo se na prvom redu

# Rješavanje modela - sile

$$\int_0^{\theta^0} \vec{f}^{\|}(\theta)d\theta = 0.$$

$\Rightarrow$

$$\frac{f_0^{\|}}{f_1^{\|}} = \frac{1 - cos(\theta_0)}{sin(\theta_0) - cos(\theta_0)\theta_0}$$

$$\vec{F}(\theta, \phi) = -\vec{F}'(\theta', \phi'),$$

$\Rightarrow$

$$f^{\|} = f_0^{\|} + f_1^{\|} = -f^{\|'} = -(f_0^{\|'} + f_1^{\|'}),$$

# Rješavanje modela - momenti

$$\int_0^{\theta_0} m^{\parallel}(\theta)d\theta = 0 \quad \Rightarrow \quad \frac{m_0^{\parallel}}{m_1^{\parallel}} = \frac{1 - cos(\theta_0)}{sin(\theta_0) - cos(\theta_0)\theta_0}$$

$$\vec{M}(\theta,\phi) + \vec{M}'(\theta',\phi') + \vec{d} \times \vec{F}(\theta,\phi) + (\vec{d'} + \vec{L}) \times \vec{F}'(\theta',\phi') = 0.$$

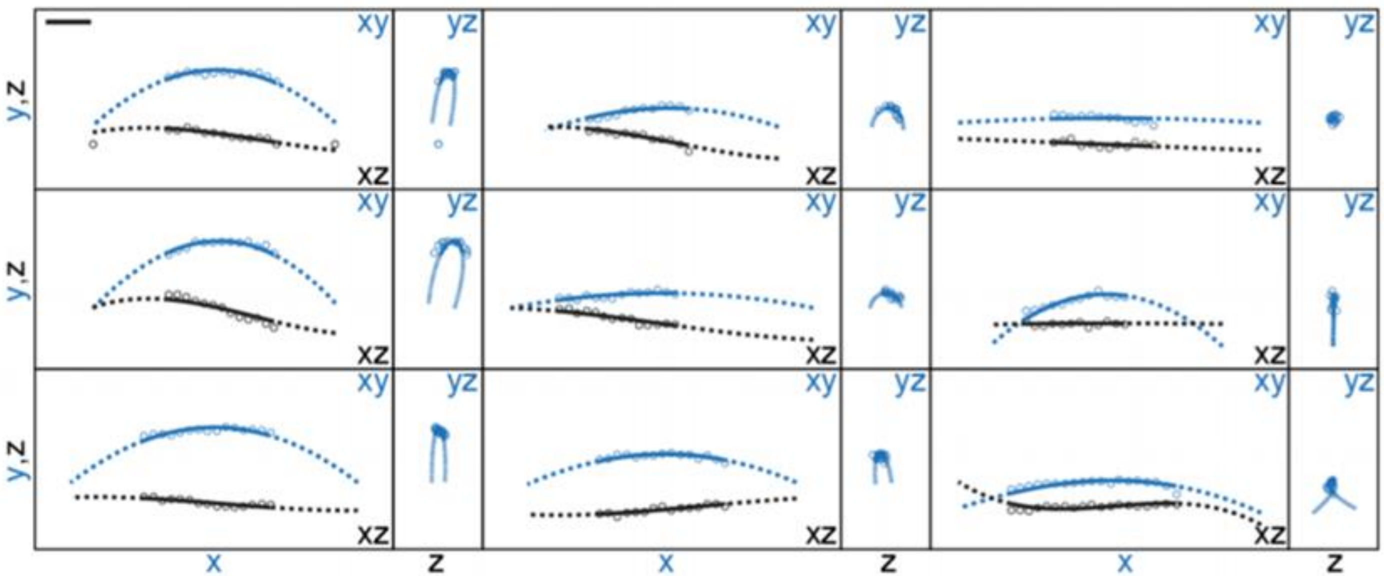
$$m^{\perp}(\theta) - m^{\perp'}(\theta) = -\frac{f^{\perp}L}{2} + \frac{1}{2}\sqrt{{f^{\perp}}^2L^2 - 4a^2 - 4af^{\perp}L},$$

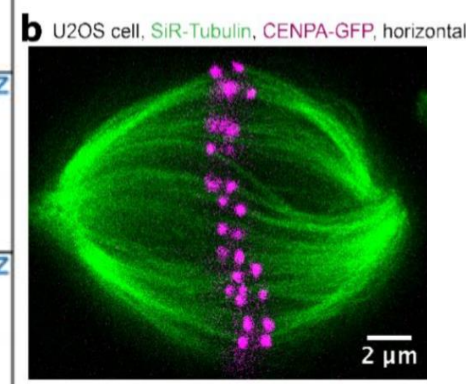
$$a = a(\theta) = 2\left|\vec{d}\right| f^{\parallel}(\theta)\theta.$$

# Oblik mikrotubula

- Statička Kirchofova jednadžba (SKJ)

$$\kappa \vec{t} \times \frac{d\vec{t}}{ds} + \tau \frac{d\phi}{ds}\vec{t} = \vec{r} \times \vec{F}_i - \vec{M}_i.$$

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Novak et al.

# Zaključak

- Od 16 koeficijenata ostaje njih 6 neovisno
- U nastavku rada treba dobiti oblike preko SKJ u usporedbi s mikroskopijom
- Steći uvid u statiku cjelokupnog vretena i kako do nje uopće dolazi

# Zahvale

Zahvaljujem se Pavin-Tolić grupi na mogućnosti izradi seminara kao i svojim mentorima :

izv. prof. dr. sc. Nenad Pavin

mag. phy. Ivana Ban