

3.

(*)

$$f: \mathbb{R}^n \rightarrow \mathbb{R}^n \quad f(x_1, x_2, \dots, x_n) = (x_n, x_{n-1}, \dots, x_2, x_1)$$

$$g: \mathbb{R}^n \rightarrow \mathbb{R}^n$$

$$f'(x) = \begin{bmatrix} 0 & 0 & \dots & 0 & 1 \\ 0 & 0 & & 1 & 0 \\ 0 & 0 & & 0 & 0 \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 1 & \dots & 0 & 0 \\ 1 & 0 & \dots & 0 & 0 \end{bmatrix}$$

$$g'(x) = \begin{bmatrix} \frac{\partial g_1}{\partial x_1}(x) & \frac{\partial g_1}{\partial x_2}(x) & \dots & \frac{\partial g_1}{\partial x_n}(x) \\ \frac{\partial g_2}{\partial x_1}(x) & \frac{\partial g_2}{\partial x_2}(x) & \dots & \frac{\partial g_2}{\partial x_n}(x) \\ \vdots & \vdots & & \vdots \\ \frac{\partial g_n}{\partial x_1}(x) & \frac{\partial g_n}{\partial x_2}(x) & \dots & \frac{\partial g_n}{\partial x_n}(x) \end{bmatrix}$$

$$(f \circ g)'(x) = f'(g(x)) \cdot g'(x) =$$

$$= \begin{bmatrix} \frac{\partial g_n}{\partial x_1}(x) & \frac{\partial g_n}{\partial x_2}(x) & \dots & \frac{\partial g_n}{\partial x_n}(x) \\ \vdots & \vdots & & \vdots \\ \frac{\partial g_2}{\partial x_1}(x) & \frac{\partial g_2}{\partial x_2}(x) & \dots & \frac{\partial g_2}{\partial x_n}(x) \\ \frac{\partial g_1}{\partial x_1}(x) & \frac{\partial g_1}{\partial x_2}(x) & \dots & \frac{\partial g_1}{\partial x_n}(x) \end{bmatrix}$$

$$\frac{\partial (f \circ g)_i}{\partial x_j}(x) = [i\text{-ti vektor, } j\text{-ti stupor}$$

gornje matrice] =

$$= \frac{\partial g_{(n+1-i)}}{\partial x_j}(x)$$

$$\boxed{\frac{\partial (f \circ g)_i}{\partial x_j} = \frac{\partial g_{(n+1-i)}}{\partial x_j}}$$

$$(g \circ f)'(x) = g'(f(x)) \cdot f'(x) =$$

$$= \begin{bmatrix} \frac{\partial g_1}{\partial x_1}(f(x)) & \frac{\partial g_1}{\partial x_2}(f(x)) & \dots & \frac{\partial g_1}{\partial x_n}(f(x)) \\ \frac{\partial g_2}{\partial x_1}(f(x)) & \frac{\partial g_2}{\partial x_2}(f(x)) & \dots & \frac{\partial g_2}{\partial x_n}(f(x)) \\ \vdots & \vdots & & \vdots \\ \frac{\partial g_n}{\partial x_1}(f(x)) & \frac{\partial g_n}{\partial x_2}(f(x)) & \dots & \frac{\partial g_n}{\partial x_n}(f(x)) \end{bmatrix}$$

$$\cdot \begin{bmatrix} 0 & 0 & \dots & 0 & 1 \\ 0 & 0 & & 1 & 0 \\ 0 & 0 & & 0 & 0 \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 1 & \dots & 0 & 0 \\ 1 & 0 & \dots & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\partial g_1}{\partial x_n}(f(x)) & \frac{\partial g_1}{\partial x_{n-1}}(f(x)) & \dots & \frac{\partial g_1}{\partial x_1}(f(x)) \\ \frac{\partial g_2}{\partial x_n}(f(x)) & \frac{\partial g_2}{\partial x_{n-1}}(f(x)) & \dots & \frac{\partial g_2}{\partial x_1}(f(x)) \\ \vdots & \vdots & & \vdots \\ \frac{\partial g_n}{\partial x_n}(f(x)) & \frac{\partial g_n}{\partial x_{n-1}}(f(x)) & \dots & \frac{\partial g_n}{\partial x_1}(f(x)) \end{bmatrix}$$

$$\frac{\partial (g \circ f)_i}{\partial x_j}(x) = [i\text{-ti redok, } j\text{-ti stupac}$$

prethodne matrice]

$$= \frac{\partial g_i}{\partial x_{(n+1-j)}}(f(x))$$