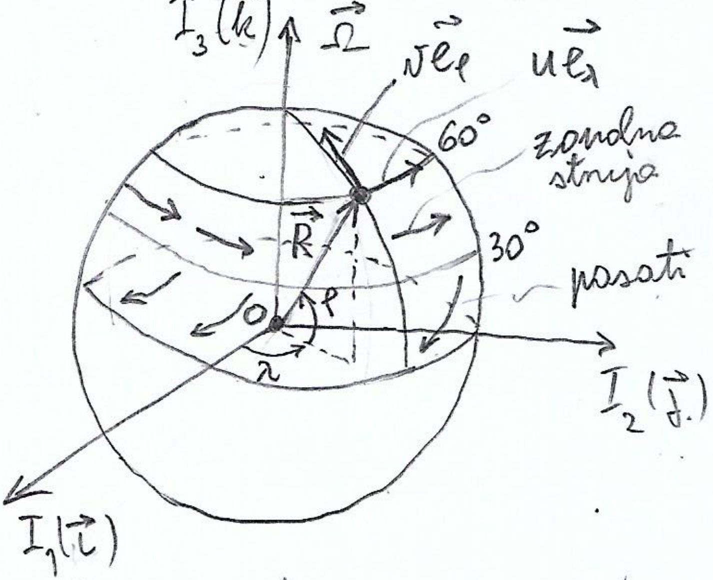
OČUVANJE ZAKRETNOG IMPULSA ATMOSFERE

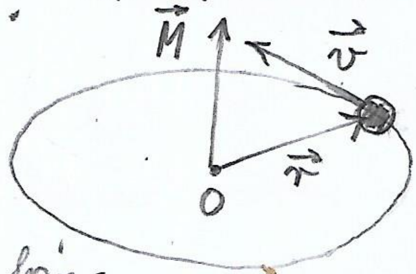
- promatramo na koji način zonalna struja umjerenih širina prenosi impuls od juga prema sjeveru



- promotrimo Zemlju u Kartezijevom $I_1I_2I_3$ sustavu (jed. vektori $\vec i, \vec j, \vec k$)
- općenita definicija zakretnog impulsa po jed. mase:

$$\vec M = \frac{\vec L}{m} = \vec r \times \vec v$$

- Pr: kamen na vretenu:



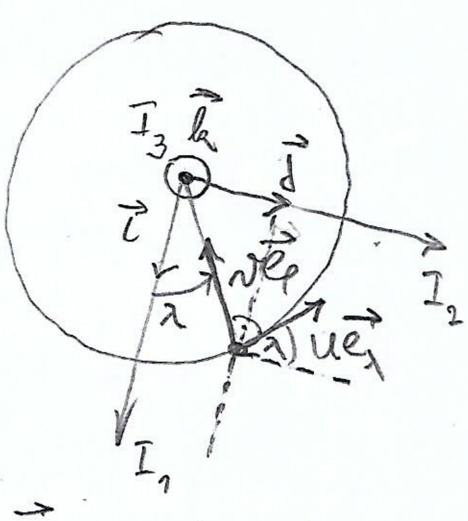
- u našem slučaju ulogu $\vec r$-a ima $\vec R$, a ulogu $\vec v$-a ima $\vec v_a = \underbrace{\vec v_r}_{\text{relativna komponenta}} + \underbrace{\vec\Omega \times \vec R}_{\text{komponenta brzine zbog rotacije Zemlje}}$

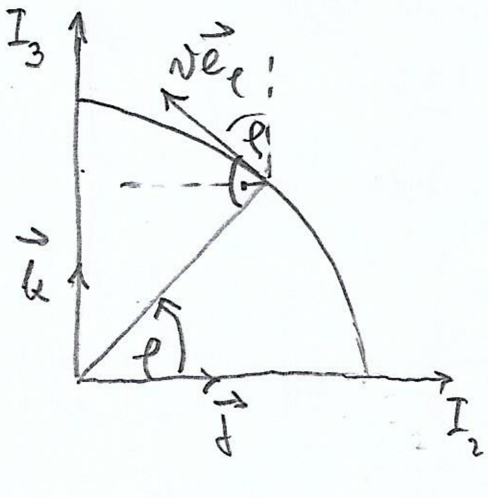
$\Rightarrow \vec M = \vec R \times \vec v_a$

- rastav $\vec R$-a u $I_1I_2I_3$ sustavu:

$$\vec R = R\cos\varphi\cos\lambda\,\vec i + R\cos\varphi\sin\lambda\,\vec j + R\sin\varphi\,\vec k$$

- kutna brzina rotacije Zemlje u $I_1I_2I_3$ $\Rightarrow \vec\Omega = \Omega\vec k$
- rel. komp. brzine u sfernom sustavu: $\vec v_r = u\vec e_\lambda + v\vec e_\varphi + \overset{\approx 0}{w\vec e_r}$
- prikaz jed. vektora $\vec e_\lambda$ i $\vec e_\varphi$ u $I_1I_2I_3$ sustavu:





$\Rightarrow \vec e_\lambda = -\sin\lambda\,\vec i + \cos\lambda\,\vec j$; $\vec e_\varphi = -\sin\varphi\cos\lambda\,\vec i - \sin\varphi\sin\lambda\,\vec j + \cos\varphi\,\vec k$

$\Rightarrow \vec v_r = (-u\sin\lambda - v\sin\varphi\cos\lambda)\vec i + (u\cos\lambda - v\sin\varphi\sin\lambda)\vec j + v\cos\varphi\,\vec k$

$$\vec{\Omega}\times\vec{R} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0 & \Omega \\ R\cos\varphi\cos\lambda & R\cos\varphi\sin\lambda & R\sin\varphi \end{vmatrix} = -\Omega R\cos\varphi\sin\lambda\,\vec{i} + \Omega R\cos\varphi\cos\lambda\,\vec{j}$$

- sada:

$$\vec{M} = \vec{R}\times\vec{v}_a = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ R\cos\varphi\cos\lambda & R\cos\varphi\sin\lambda & R\sin\varphi \\ -(u+R\Omega\cos\varphi)\sin\varphi - v\sin\varphi\cos\lambda & (u+\Omega R\cos\varphi)\cos\lambda + v\sin\varphi\cos\lambda & v\cos\varphi \end{vmatrix}$$

$$\Rightarrow M = \vec{i}\left\{R\cos\varphi\sin\lambda\, v\cos\varphi - R\sin\varphi\left[(u+\Omega R\cos\varphi)\cos\lambda + v\sin\varphi\cos\lambda\right]\right\} -$$
$$-\vec{j}\left\{R\cos\varphi\cos\lambda\, v\cos\varphi - R\sin\varphi\left[-(u+R\Omega\cos\varphi)\sin\varphi - v\sin\varphi\cos\lambda\right]\right\} +$$
$$+\vec{k}\left\{R\cos\varphi\cos\lambda\left[(u+\Omega R\cos\varphi)\cos\lambda + v\sin\varphi\cos\lambda\right] -\right.$$
$$\left. - R\cos\varphi\sin\lambda\left[-(u+R\Omega\cos\varphi)\sin\varphi - v\sin\varphi\cos\lambda\right]\right\}$$

<u>DZ</u> $\Rightarrow$ srediti gornji račun!

$$\Rightarrow \vec{M} = [\ldots\ldots]\,\vec{i} + [\ldots\ldots]\,\vec{j} + \left[(u+\Omega R\cos\varphi)R\cos\varphi\right]\vec{k} =$$
$$= M_1\vec{i} + M_2\vec{j} + M_3\vec{k}$$

- komponente M_1 i M_2 nemaju projekcije prema sjeveru, zanima nas samo M_3!

$$\Rightarrow M_3 = R\cos\varphi\,(\underbrace{u+\Omega R\cos\varphi}_{v_x}) = R\cos\varphi\, v_x$$
$\Rightarrow$ samo zonalna komp. brzine igra ulogu u prijenosu impulsa!

- 2. Newtonov zakon u formi zakretnog impulsa (u sustavu s vert. koord. tlaka)

$$\frac{dM_3}{dt} = R\cos\varphi\frac{dv_x}{dt} = R\cos\varphi\left[\underbrace{-\left(\frac{\partial\Phi}{\partial x}\right)_p}_{\text{sila grad. tlaka}} \underbrace{- g\frac{\partial\tau_E^x}{\partial p}}_{\text{vert. smicanje (trenje)}}\right]$$

(u daljnjem uvodu ispuštamo indeks 3)

- digresija: u xyz sustavu član smicanja glasi:

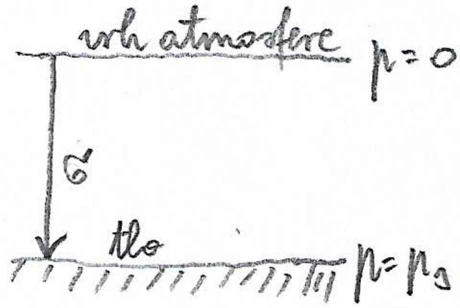
$$S = \frac{1}{\rho}\frac{\partial\tau_E^x}{\partial z} = \left\{\frac{\partial p}{\partial z} = -\rho g \Rightarrow \frac{1}{\partial z} = -\frac{\rho g}{\partial p}\right\} = -g\frac{\partial\tau_E^x}{\partial p}$$ ✓✓

$\Rightarrow \boxed{\frac{dM}{dt} = -R\cos\varphi\left[\left(\frac{\partial\Phi}{\partial x}\right)_p + g\,\frac{\partial \tau_E^{x}}{\partial p}\right]}$ jednadžba s notacijom iz Holton-a

- lakše je raditi u koord. sustavu gdje je vert. koordinata σ ...

σ KOORDINATA

- donja granica koincidira s površinom



- def: $\boxed{\sigma = \frac{p}{p_s}}$; p_s ... površinski tlak

$p_s = p_s(x,y,t)$

$\Rightarrow \sigma \in [0,\ 1]$ (0 – vrh, 1 – tlo)

- vertikalna brzina: $\dot{\sigma} = \frac{d\sigma}{dt}$

$\Rightarrow$ pri tlu: $\sigma_s = 1 \Rightarrow \dot{\sigma}_s = 0$

- transformacija iz sustava s tlakom u sustav s σ koord:
- sila gradijenta tlaka:
  - općenito, za generaliziranu koordinatu vrijedi:

$$\left(\frac{\partial a}{\partial x}\right)_\xi = \left(\frac{\partial a}{\partial x}\right)_\eta + \frac{\partial \xi}{\partial \eta}\left(\frac{\partial \eta}{\partial x}\right)_\xi \frac{\partial a}{\partial \xi}$$

- ovdje: $\left.\begin{array}{l} \xi = \sigma \\ \eta = p \\ a = \Phi \end{array}\right\}$ $\left(\frac{\partial \Phi}{\partial x}\right)_\sigma = \left(\frac{\partial \Phi}{\partial x}\right)_p + \frac{\partial \sigma}{\partial p}\left(\frac{\partial p}{\partial x}\right)_\sigma \frac{\partial \Phi}{\partial \sigma}$ (*)

$$\frac{\partial \sigma}{\partial p} = \frac{1}{p_s}\frac{\partial p}{\partial p} = \frac{1}{p_s}$$

$$\Rightarrow \left(\frac{\partial \Phi}{\partial x}\right)_\sigma = \left(\frac{\partial \Phi}{\partial x}\right)_p + \frac{1}{p_s}\left(\frac{\partial p}{\partial x}\right)_\sigma \frac{\partial \Phi}{\partial \sigma} = \left(\frac{\partial \Phi}{\partial x}\right)_p + \frac{1}{p_s}\left[\frac{\partial(\sigma p_s)}{\partial x}\right]_\sigma \frac{\partial \Phi}{\partial \sigma}$$

$$\Rightarrow \left(\frac{\partial \Phi}{\partial x}\right)_\sigma = \left(\frac{\partial \Phi}{\partial x}\right)_p + \sigma\left(\frac{\partial \ln p_s}{\partial x}\right)\frac{\partial \Phi}{\partial \sigma}$$

$$\Rightarrow \left(\frac{\partial \phi}{\partial x}\right)_\sigma = \left(\frac{\partial \phi}{\partial x}\right)_p + \sigma \left(\frac{\partial \ln p_s}{\partial x}\right)\left(\frac{\partial \phi}{\partial \sigma}\right)$$

- bilo koja druga varijabla ide ANALOGNO:

$$\nabla_\sigma(\ ) = \nabla_p(\ ) + \sigma \nabla \ln p_s \frac{\partial(\ )}{\partial \sigma}$$

$$\Rightarrow \nabla_p(\ ) = \nabla_\sigma(\ ) - \sigma \nabla \ln p_s \frac{\partial(\ )}{\partial \sigma}$$

- jdba gibanja: $\frac{d\vec{V}}{dt} + f\vec{k}\times\vec{V} = -\nabla_p \phi$

$$\Rightarrow \frac{d\vec{V}}{dt} + f\vec{k}\times\vec{V} = -\nabla_\sigma \phi + \sigma \nabla \ln p_s \frac{\partial \phi}{\partial \sigma}$$

$$\boxed{\frac{d}{dt} = \frac{\partial}{\partial t} + \vec{V}\cdot\nabla_\sigma + \dot{\sigma}\frac{\partial}{\partial \sigma}}$$

- jdba kontinuiteta: $\nabla_p \cdot \vec{V} + \frac{\partial \omega}{\partial p} = 0$

$$\nabla_p \cdot \vec{V} = \nabla_\sigma \cdot \vec{V} - \sigma \nabla \ln p_s \cdot \frac{\partial \vec{V}}{\partial \sigma}$$

$$\frac{\partial}{\partial p} = \frac{\partial}{\partial(\sigma p_s)} = \frac{1}{p_s}\frac{\partial}{\partial \sigma} \Rightarrow \frac{\partial \omega}{\partial p} = \frac{1}{p_s}\frac{\partial \omega}{\partial \sigma}$$

$$\Rightarrow \nabla_p \cdot \vec{V} + \frac{1}{p_s}\frac{\partial \omega}{\partial \sigma} = 0 \,/\, p_s \Rightarrow p_s(\nabla_p \cdot \vec{V}) + \frac{\partial \omega}{\partial \sigma} = 0 \quad (*)$$

$\Rightarrow$ ŽELIMO $\dot{\sigma}$, a ne ω!

- vertikalna σ brzina: $\dot{\sigma} = \frac{d\sigma}{dt} = \frac{\partial \sigma}{\partial t} + \vec{V}\cdot\nabla_p \sigma + \omega \frac{\partial \sigma}{\partial p}$

$$\Rightarrow \dot{\sigma} = \frac{\partial}{\partial t}\left(\frac{p}{p_s}\right) + \vec{V}\cdot\nabla_p\left(\frac{p}{p_s}\right) + \omega\frac{\partial}{\partial p}\left(\frac{p}{p_s}\right) =$$

$$= \frac{-p\frac{\partial p_s}{\partial t}}{p_s^2} + \vec{V}\cdot\frac{-p\nabla_p p_s}{p_s^2} + \frac{\omega}{p_s} = \underbrace{-\frac{\sigma}{p_s}\Big(\underset{(1)}{\frac{\partial p_s}{\partial t}} + \underset{(2)}{\vec{V}\cdot\nabla_p p_s}\Big) + \underset{(3)}{\frac{\omega}{p_s}}}_{\boxed{1}}$$

- dobiveno je:

(3) član: $\omega \frac{\partial \sigma}{\partial p} = \frac{\omega}{p_s}$

(2) član: $\vec{V}\cdot\nabla_p \sigma = -\frac{\sigma}{p_s}\vec{V}\cdot\nabla_p p_s$

(1) član: $\frac{\partial \sigma}{\partial t} = -\frac{\sigma}{p_s}\frac{\partial p_s}{\partial t}$

- sada diferenciramo izraz $\boxed{1}$ po σ $\Rightarrow$ dobije se

$$\frac{\partial \dot\sigma}{\partial \sigma} = -\frac{1}{p_s}\left(\frac{\partial p_s}{\partial t} + \vec{v}\cdot\nabla p_s\right) - \frac{\sigma}{p_s}\frac{\partial}{\partial \sigma}\left(\frac{\partial p_s}{\partial t} + \vec{v}\cdot\nabla p_s\right) + \frac{1}{p_s}\frac{\partial \omega}{\partial \sigma}$$

(*) daje: $p_s\left(\nabla_\sigma\cdot\vec{v} - \sigma\left(\nabla \ln p_s\right)\cdot\frac{\partial \vec{V}}{\partial \sigma}\right) + \frac{\partial \omega}{\partial \sigma} = 0$

$\vdots$

$$\Rightarrow \frac{\partial \dot\sigma}{\partial \sigma} = -\frac{1}{p_s}\frac{\partial p_s}{\partial t} - \frac{1}{p_s}\vec{V}\cdot\nabla p_s - \nabla\cdot\vec{V} \quad / \cdot p_s$$

$$\Rightarrow \boxed{\frac{\partial p_s}{\partial t} + \nabla\cdot\left(p_s\vec{V}\right) + p_s\frac{\partial \dot\sigma}{\partial \sigma} = 0} \rightarrow \text{NOVA JDŽBA KONT.} \quad \boxed{2}$$

- jdžba hidrostatike u σ sustavu: $\frac{\partial \phi}{\partial \sigma} = -\frac{RT}{\sigma} = -\frac{R\theta}{\sigma}\left(\frac{p}{p_0}\right)^{\varkappa}$

- TD jdžba u σ sustavu: $\frac{\partial \theta}{\partial t} + \vec{v}\cdot\nabla\theta + \dot\sigma\frac{\partial \theta}{\partial \sigma} = \frac{Q}{c_p}\frac{\theta}{T}$

- transformacija jdžbe zakretnog impulsa u σ sustav:

$$\frac{dM}{dt} = -R\cos\varphi\left(\frac{\partial \phi}{\partial x} + g\frac{\partial \tau_E^x}{\partial p}\right)$$

- dobile: $\left(\frac{\partial \phi}{\partial x}\right)_p = \left(\frac{\partial \phi}{\partial x}\right)_\sigma - \sigma\left(\frac{\partial \ln p_s}{\partial x}\right)\left(\frac{\partial \phi}{\partial \sigma}\right)$ $\rightarrow$ MAJA

$$\Rightarrow \left(\frac{\partial}{\partial t} + \vec{V}\cdot\nabla_\sigma + \dot\sigma\frac{\partial}{\partial \sigma}\right)M = -R\cos\varphi\left[\left(\frac{\partial \phi}{\partial x}\right)_\sigma - \frac{\cancel{\sigma}}{p_s}\frac{\partial p_s}{\partial x}\left(-\frac{RT}{\cancel{\sigma}}\right) + g\frac{\partial \tau_E^x}{\partial (p_s\cdot\sigma)}\right]$$

$$\Rightarrow \boxed{\left(\frac{\partial}{\partial t} + \vec{V}\cdot\nabla_\sigma + \dot\sigma\frac{\partial}{\partial \sigma}\right)M = -R\cos\varphi\left[\left(\frac{\partial \phi}{\partial x}\right)_\sigma + \frac{RT}{p_s}\frac{\partial p_s}{\partial x} + \frac{g}{p_s}\frac{\partial \tau_E^x}{\partial \sigma}\right]} - \boxed{3}$$

- SADA: $\underbrace{\boxed{2}\cdot M + \boxed{3}\cdot p_s}$

$\rightarrow$ MAJA $\quad \boxed{4}$

$$\boxed{\frac{\partial}{\partial t}(Mp_s) + \nabla\cdot\left(Mp_s\vec{V}\right) + \frac{\partial\left(p_s M\dot\sigma\right)}{\partial \sigma} = -R\cos\varphi\left[p_s\frac{\partial \phi}{\partial x} + RT\frac{\partial p_s}{\partial x}\right] - gR\cos\varphi\frac{\partial \tau_E^x}{\partial \sigma}}$$

$\hookrightarrow$ da se dobije budžet zonalno usrednjenog zakretnog impulsa ovo treba zonalno usrednjiti

- u novoj sfernih koordinata: $\vec{A} = a\vec{i} + b\vec{j}$

$$\nabla\cdot\vec{A} = \frac{1}{r\cos\varphi}\left[\frac{\partial a}{\partial\lambda} + \frac{\partial(b\cos\varphi)}{\partial\varphi}\right] \rightarrow \text{Bronštejn '75 (str. 630)}$$

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$$\Rightarrow \boxed{\nabla\cdot(p_s M\vec{V}) = \frac{1}{R\cos\varphi}\left[\frac{\partial(p_s M u)}{\partial\lambda} + \frac{\partial(p_s M v\cos\varphi)}{\partial\varphi}\right]} \quad \boxed{5}\ (*)$$

- član: $p_s\frac{\partial\phi}{\partial x} + RT\frac{\partial p_s}{\partial x} = p_s\frac{\partial}{\partial x}(\phi - RT) + \frac{\partial}{\partial x}(p_s RT)$

$$\phi - RT = \phi + \sigma\frac{\partial\phi}{\partial\sigma} = \frac{\partial(\sigma\phi)}{\partial\sigma}$$

- budući da p_s ne ovisi o σ:

$$\boxed{p_s\frac{\partial\phi}{\partial x} + RT\frac{\partial p_s}{\partial x} = \frac{\partial}{\partial\sigma}\left(p_s\sigma\frac{\partial\phi}{\partial x}\right) + \frac{\partial}{\partial x}(p_s RT)} \quad \boxed{6}$$

- uvrstimo 5 i 6 u 4 i zonalno usrednjimo (po λ):

0 → NEMA PROMJENE ZONALNE STRUJE PO X OSI!

$$\frac{\partial}{\partial t}(Mp_s) + \frac{1}{R\cos\varphi}\cancel{\frac{\partial}{\partial\lambda}(p_s M u)} + \frac{1}{R\cos\varphi}\frac{\partial}{\partial\varphi}(p_s M v\cos\varphi) + \frac{\partial}{\partial\sigma}(p_s M\dot{\sigma}) =$$
$$= -R\cos\varphi\frac{\partial}{\partial\sigma}\left(p_s\sigma\frac{\partial\phi}{\partial x}\right) - R\cos\varphi\cancel{\frac{\partial}{\partial x}(p_s RT)} - gR\cos\varphi\frac{\partial\tau_E^x}{\partial\sigma}$$

to ne piši!

0 NEMA DIFF. ZAGRIJAVANJA PO X OSI (OD W PREMA E)!

$$\Rightarrow \boxed{7}\ \boxed{\frac{\partial}{\partial t}(\overline{p_s M}) = \underbrace{-\frac{1}{\cos\varphi}\frac{\partial}{\partial y}(\overline{p_s M v\cos\varphi})}_{\text{konvergencija horizontalnog fluksa zakretnog impulsa}} \underbrace{-\frac{\partial}{\partial\sigma}\left[\overline{p_s M\dot{\sigma}} + gR\cos\varphi(\overline{\tau_E^x}) + R\cos\varphi\,\overline{\sigma p_s\frac{\partial\phi}{\partial x}}\right]}_{\text{konvergencija vertikalnog fluksa zakretnog impulsa}}}.$$

- integriramo 7 od površine ($\sigma = 1$) do vrha atmosfere ($\sigma = 0$) uz RU $\dot{\sigma} = 0$ za $\sigma = 0, 1$

- pomnožimo 7 sa $\frac{1}{g}$ da bismo zadnji član na desno u 7 mogli pisati kao:

$$\overline{R\cos\varphi\,\sigma p_s\frac{1}{g}\frac{\partial\phi}{\partial x}} = \overline{R\cos\varphi\,\sigma p_s\frac{\partial h}{\partial x}}$$

$$\Rightarrow \int_0^1\frac{1}{g}\frac{\partial}{\partial t}(\overline{p_s M})\,d\sigma = -\frac{1}{g\cos\varphi}\int_0^1\frac{\partial}{\partial y}(\overline{p_s M v\cos\varphi})\,d\sigma - \frac{1}{g}\left[\cancel{(\overline{p_s M\dot{\sigma}})_{\sigma=1}}^{0} - \cancel{(\overline{p_s M\dot{\sigma}})_{\sigma=0}}^{0} +\right.$$
$$\left. + gR\cos\varphi(\overline{\tau_E^x})_{\sigma=1} - gR\cos\varphi\cancel{(\overline{\tau_E^x})_{\sigma=0}}^{0}\right] - \left[\overline{R\cos\varphi\,\sigma p_s\frac{\partial h}{\partial x}}\right]_{\sigma=0}^{\sigma=1}$$

$$\Rightarrow \boxed{\int_0^1 g^{-1}\frac{\partial}{\partial t}\overline{p_s M}\, d\sigma = -(g\cos\varphi)^{-1}\int_0^1 \frac{\partial}{\partial y}\left(\overline{p_s M v}\cos\varphi\right) d\sigma - R\cos\varphi\left[\left(\overline{\tau_E^{x}}\right)_{\sigma=1} + \overline{p_s \frac{\partial h}{\partial x}}\right]}$$

(1) (2) (3)

[18] bilanca zakretnog impulsa po jediničnoj meridionalnoj širini od tla do vrha atmosfere

- zonalni stres iščezava na vrhu!

$(1) = -\frac{1}{g\cos\varphi}\int_0^1 \frac{\partial}{\partial y}\left(\overline{p_s M v}\cos\varphi\right) d\sigma$ ... konvergencija meridionalnog toka zakretnog impulsa zbog gibanja velike skale

$(2) = -R\cos\varphi\left(\overline{\tau_E^{x}}\right)_{\sigma=1}$ ... tok zbog turbulentnih napetosti male skale na površini

$(3) = -R\cos\varphi\,\overline{p_s \frac{\partial h}{\partial x}}$ ... površinski zakretni impuls tlaka

- članovi (2) i (3) odgovorni su za prijenos količine gibanja po vertikali i to od tla prema atmosferi u tropima te od atmosfere prema tlu u umjerenim širinama
- član (1) odgovoran je za prijenos količine gibanja od ekvatora prema polu ⇒ TO NAS ZANIMA!
- to ćemo analizirati t.d. ćemo strujanje podijeliti na zonalni srednjak i perturbacije:

$$\overline{M} = \overline{M} + M' = (\Omega R\cos\varphi + \overline{u} + u')R\cos\varphi$$

$$p_s v = (\overline{p_s v}) + (p_s v)'$$

- sada meridionalni tok postaje:

$$(\overline{p_s M v}) = \Big[\underbrace{\Omega R\cos\varphi\,\overline{p_s v}}_{(1.1.)} + \underbrace{\overline{u}\,\overline{p_s v}}_{(1.2.)} + \underbrace{\overline{u'(p_s v')}}_{(1.3.)}\Big]R\cos\varphi$$

(1.1.) meridionalni Ω-tok impulsa

(1.2.) meridionalni pomak

(1.3.) meridionalni eddy-tok impulsa

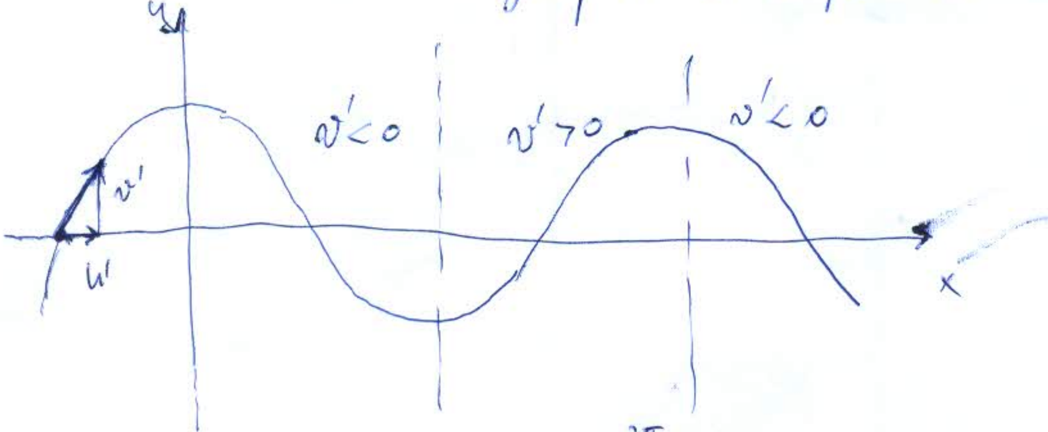
- član (1.2.) značajan je u tropima dok je u umjerenim širinama zanemariv u odnosu na (1.3.)

- član (1.1.) ne pridonosi vertikalno integriranom toku (suprotni smjerovi gornje ćelije)

- stoga je kistom samo član (1.3) pa (1) postoje:

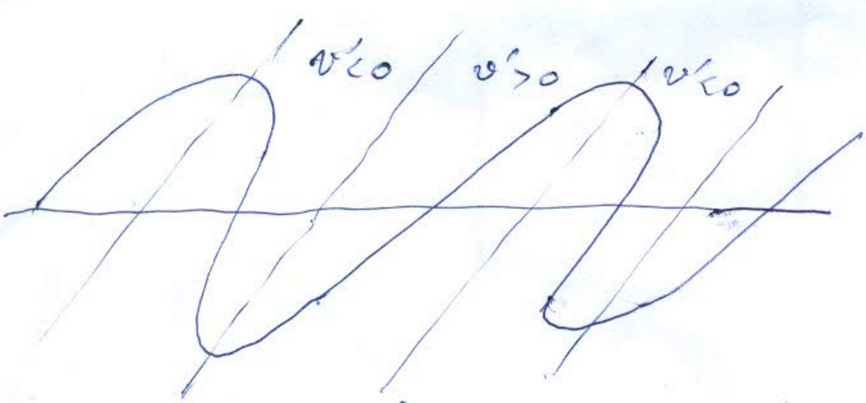
$$\int_0^1 \overline{p_s M v}\, d\sigma \approx \int_0^1 \overline{u'(p_s v')} R\cos\varphi\, d\sigma \approx \int_0^1 R\cos\varphi\, \overline{p_s}\, \overline{u'v'}\, d\sigma$$

- došli smo na to da mjera za prijenos impulsa $\overline{u'v'}$!
- kako sada valovi mogu prenositi impuls od S prema N?



- za ovako postavljen val: $\int_0^{2\pi} u'v'\, d\lambda = 0$

- no, valovi su na NH zbog kvazigeostrofičnosti nagnuti od SW prema NE



- zbog toga se impuls prema N prenosi dugotrajnije i u većoj količini nego prema S:
$$\int_0^{2\pi} u'v'\, d\lambda > 0$$

⇒ dakle, ∃ prijenos impulsa prema N što smanjuje smicanje napetosti