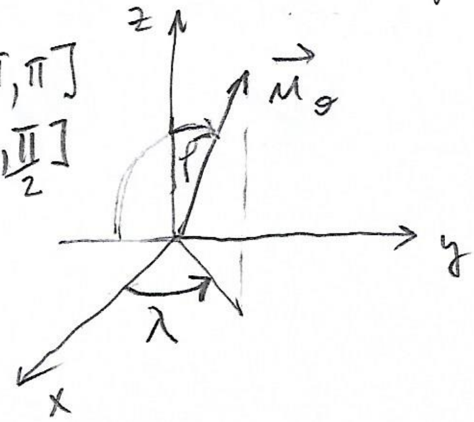
- izentropske plohe u prostoru mogu zauzeti bilo kakav položaj t.d. i jedinični vektor $\vec{n}_\theta$ u prostoru može zauzeti bilo kakvu orijentaciju

$\lambda \in [-\pi, \pi]$

$\varphi \in [0, \frac{\pi}{2}]$



- ako iskoristimo sferne koordinate za opis $\vec{n}_\theta$ u KKS-u:

$$\Rightarrow \vec{n}_\theta = \sin\varphi\cos\lambda\,\vec{i} + \sin\varphi\sin\lambda\,\vec{j} + \cos\varphi\,\vec{k}$$

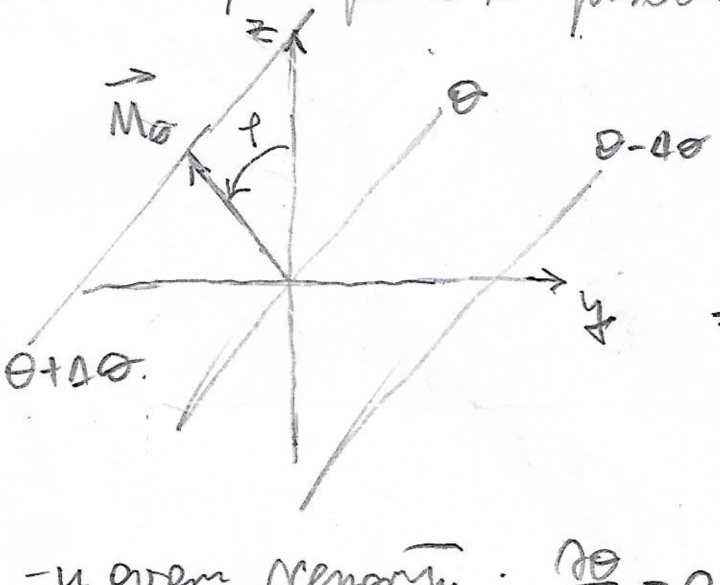
- uz ~~pp~~ adijabatičnosti $\Rightarrow \frac{d\theta}{dt} = 0$

$$\Rightarrow F = -(\sin\varphi\cos\lambda\,\vec{i} + \sin\varphi\sin\lambda\,\vec{j} + \cos\varphi\,\vec{k}) \cdot \Big[\frac{\partial\theta}{\partial x}\Big(\frac{\partial u}{\partial x}\vec{i} + \frac{\partial u}{\partial y}\vec{j} + \frac{\partial u}{\partial z}\vec{k}\Big) +$$
$$+ \frac{\partial\theta}{\partial y}\Big(\frac{\partial v}{\partial x}\vec{i} + \frac{\partial v}{\partial y}\vec{j} + \frac{\partial v}{\partial z}\vec{k}\Big) + \frac{\partial\theta}{\partial z}\Big(\frac{\partial w}{\partial x}\vec{i} + \frac{\partial w}{\partial y}\vec{j} + \frac{\partial w}{\partial z}\vec{k}\Big)\Big]$$

$$\Rightarrow F = -\sin\varphi\cos\lambda\Big(\frac{\partial\theta}{\partial x}\frac{\partial u}{\partial x} + \frac{\partial\theta}{\partial y}\frac{\partial v}{\partial x} + \frac{\partial\theta}{\partial z}\frac{\partial w}{\partial x}\Big) -$$
$$- \sin\varphi\sin\lambda\Big(\frac{\partial\theta}{\partial x}\frac{\partial u}{\partial y} + \frac{\partial\theta}{\partial y}\frac{\partial v}{\partial y} + \frac{\partial\theta}{\partial z}\frac{\partial w}{\partial y}\Big) -$$
$$- \cos\varphi\Big(\frac{\partial\theta}{\partial x}\frac{\partial u}{\partial z} + \frac{\partial\theta}{\partial y}\frac{\partial v}{\partial z} + \frac{\partial\theta}{\partial z}\frac{\partial w}{\partial z}\Big)$$

- promotrimo slijedeći scenarij $\Rightarrow$ neka je $\lambda = -\frac{\pi}{2}$ i $\varphi \in [0, \frac{\pi}{2}]$

$\Rightarrow$ izentropske plohe su paralelne s osi x $\Rightarrow$ $\boxed{\text{FG U VERTIKALNOJ RAVNINI}}$!!!



$\sin\lambda = -1$	$\sin\varphi \geq 0$
$\cos\lambda = 0$	$\cos\varphi \geq 0$

$$\Rightarrow F = \sin\varphi\Big(\frac{\partial\theta}{\partial x}\frac{\partial u}{\partial y} + \frac{\partial\theta}{\partial y}\frac{\partial v}{\partial y} + \frac{\partial\theta}{\partial z}\frac{\partial w}{\partial y}\Big)$$
$$- \cos\varphi\Big(\frac{\partial\theta}{\partial x}\frac{\partial u}{\partial z} + \frac{\partial\theta}{\partial y}\frac{\partial v}{\partial z} + \frac{\partial\theta}{\partial z}\frac{\partial w}{\partial z}\Big)$$

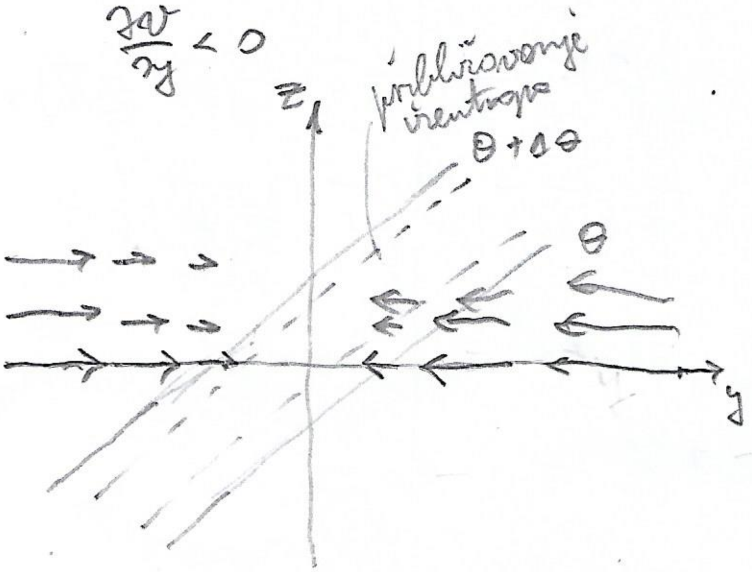
- u ovom scenariju: $\frac{\partial\theta}{\partial x} = 0$; $\frac{\partial\theta}{\partial y} < 0$; $\frac{\partial\theta}{\partial z} > 0$

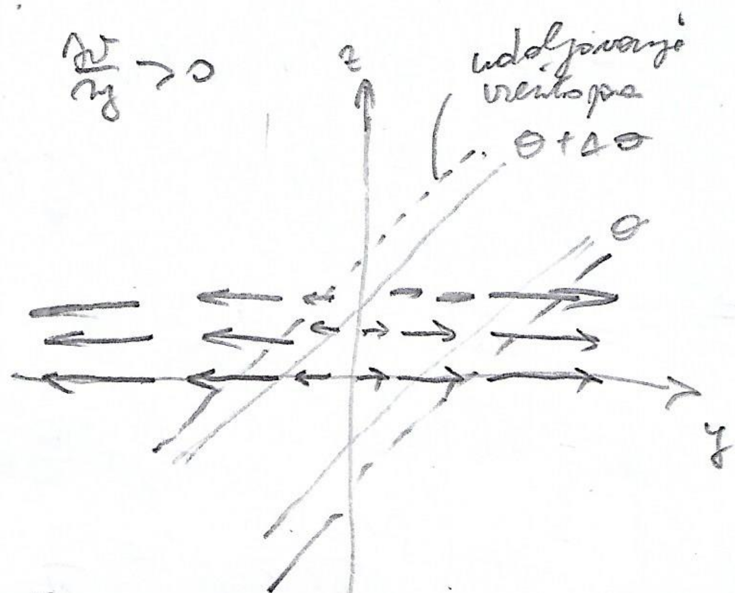
$$\Rightarrow \frac{\partial\theta}{\partial y} = -\Big|\frac{\partial\theta}{\partial y}\Big|$$

$$\Rightarrow F = \sin\varphi\left(\underset{Ⓐ}{-\left|\frac{\partial\theta}{\partial y}\right|\frac{\partial v}{\partial y}} + \underset{Ⓑ}{\frac{\partial\theta}{\partial z}\frac{\partial w}{\partial y}}\right) - \cos\varphi\left(\underset{Ⓒ}{-\left|\frac{\partial\theta}{\partial y}\right|\frac{\partial v}{\partial z}} + \underset{Ⓓ}{\frac{\partial\theta}{\partial z}\frac{\partial w}{\partial z}}\right)$$

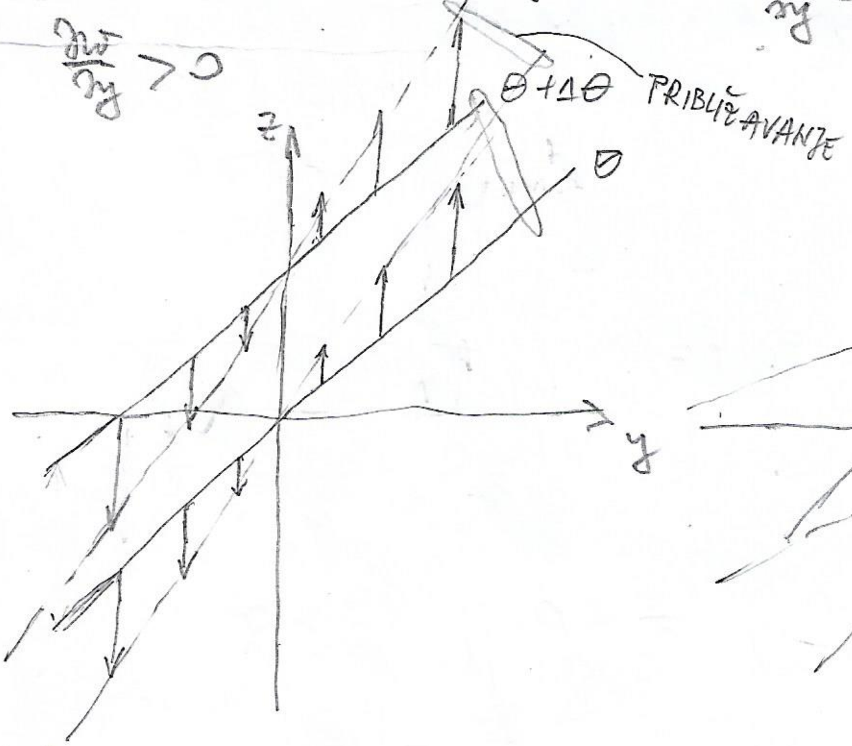
- članovi Ⓐ i Ⓓ ⇒ konfluencija, divergencija
- članovi Ⓑ i Ⓒ ⇒ tilting članovi
- analiza pojedinih članova:

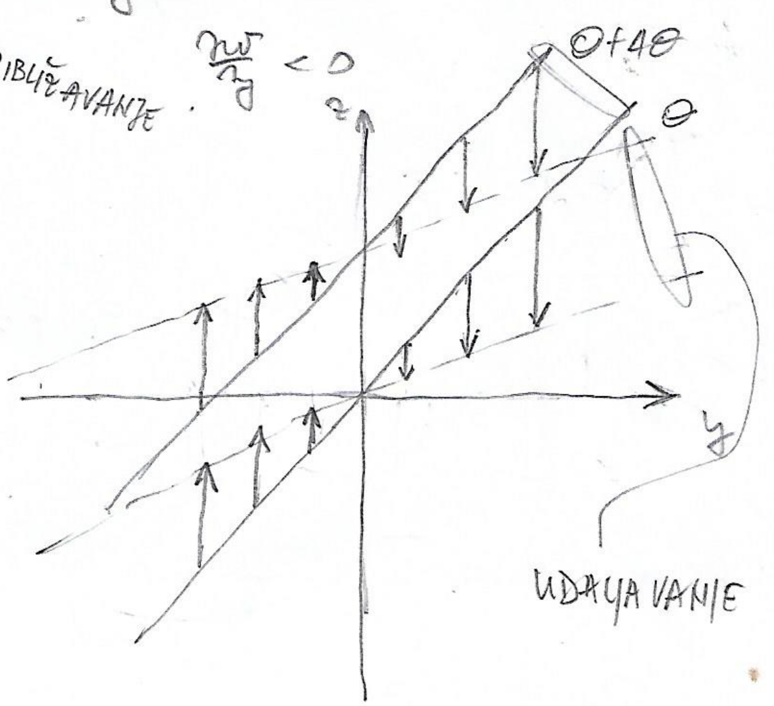
$$Ⓐ = -\left|\frac{\partial\theta}{\partial y}\right|\frac{\partial v}{\partial y} \sim -\frac{\partial v}{\partial y}\begin{cases} >0 \text{ za } \frac{\partial v}{\partial y}<0 \Rightarrow F>0 \Rightarrow FG \\ <0 \text{ za } \frac{\partial v}{\partial y}>0 \Rightarrow F<0 \Rightarrow FL \end{cases}$$



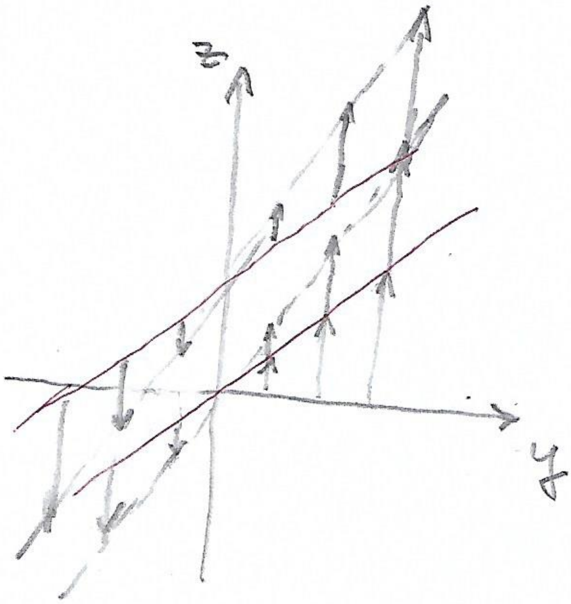


$$Ⓑ = \frac{\partial\theta}{\partial z}\frac{\partial w}{\partial y} \sim \frac{\partial w}{\partial y}\begin{cases} >0 \text{ za } \frac{\partial w}{\partial y}>0 \Rightarrow F>0 \Rightarrow FG \\ <0 \text{ za } \frac{\partial w}{\partial y}<0 \Rightarrow F<0 \Rightarrow FL \end{cases}$$

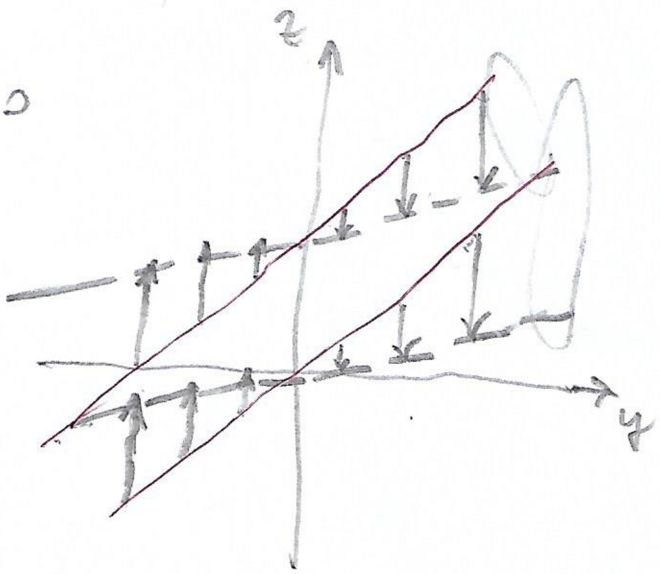




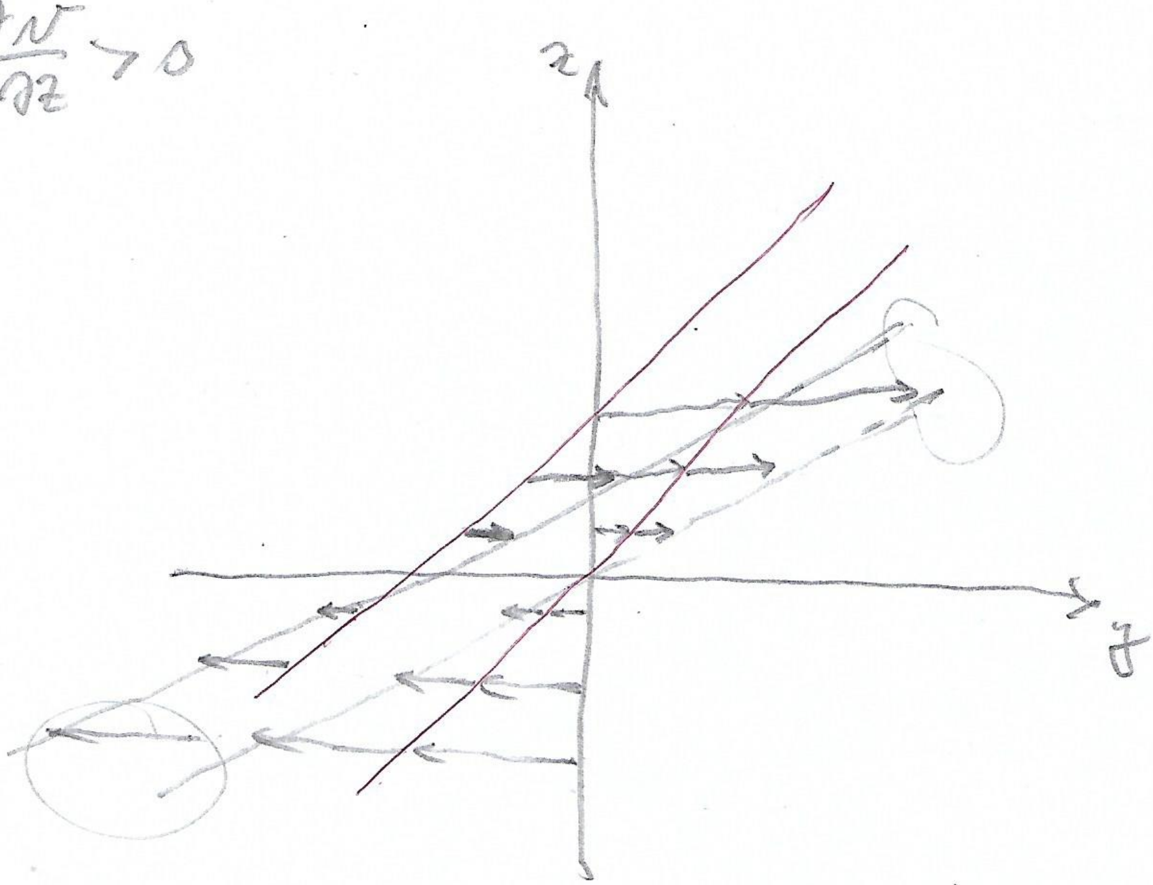
$\frac{\partial v}{\partial y} > 0$



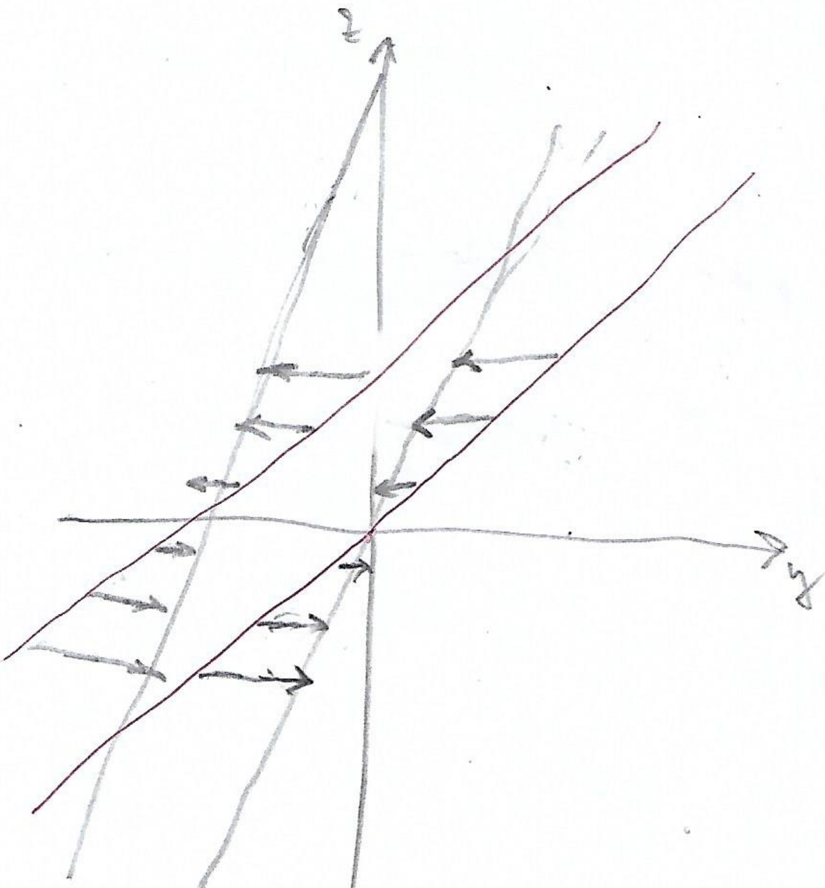
$\frac{\partial v}{\partial y} < 0$



$\frac{\partial v}{\partial z} > 0$



$\frac{\partial v}{\partial z} < 0$



⇒ članovi C i D za DZ !

- to je bila vertikalna ravnina
- ako želimo proučavati FG u HOR. ravnini, moramo se upoznati s diferencijalnim svojstvima hor. vjetra!

- neka je $u = u(x,y)$ i $v = v(x,y)$
- tada, za ishodište Kartezijevog sustava $(x_0, y_0) = (0,0)$ vrijedi T. razvoj:

$$u(x,y) = u_0 + \left(\frac{\partial u}{\partial x}\right)_0 x + \left(\frac{\partial u}{\partial y}\right)_0 y + \dots$$

$$v(x,y) = v_0 + \left(\frac{\partial v}{\partial x}\right)_0 x + \left(\frac{\partial v}{\partial y}\right)_0 y + \dots$$

zanemarujemo članove višeg reda jer razmatramo dovoljno blizu ishodišta

- prostorne derivacije hor. vjetra se mogu kombinirati na sljedeći način:

(1) $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = \nabla_H \cdot \vec{V} = \delta$ ⇒ hor. divergencija

(2) $\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = \vec{k} \cdot (\nabla \times \vec{V}) = \zeta$ ⇒ vert. komp. rel. vrtložnosti

(3) $\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} = D_1$ ⇒ deformacija rastezanja

(4) $\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} = D_2$ ⇒ deformacija smicanja

- koristeći gornje pojmove, pojedine derivacije možemo složiti kao:

(1)+(3) ⇒ $\frac{\partial u}{\partial x} = \frac{1}{2}(\delta + D_1)$

(1)-(3) ⇒ $\frac{\partial v}{\partial y} = \frac{1}{2}(\delta - D_1)$

(2)+(4) ⇒ $\frac{\partial v}{\partial x} = \frac{1}{2}(\zeta + D_2)$

(2)-(4) ⇒ $\frac{\partial u}{\partial y} = -\frac{1}{2}(\zeta - D_2)$

to u T. razvoj

$$\Rightarrow u(x,y) = u_0 + \frac{1}{2}\delta x - \frac{1}{2}\zeta y + \frac{1}{2}D_1 x + \frac{1}{2}D_2 y$$

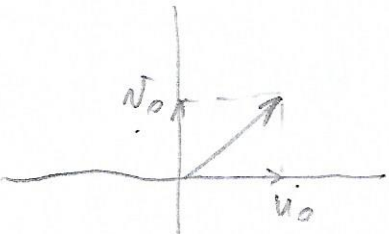
$$v(x,y) = v_0 + \frac{1}{2}\delta y + \frac{1}{2}\zeta x + \frac{1}{2}D_2 x - \frac{1}{2}D_1 y$$

– spec. slučajevi po članovima:

1) $\delta, \zeta, D_1, D_2 = 0$; $u_0, v_0 \neq 0$

$\Rightarrow u = u_0$
$v = v_0$

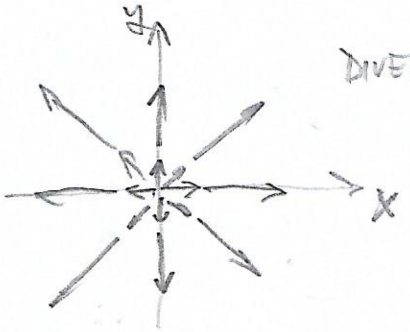
npr. $u_0, v_0 > 0$



$\Rightarrow$ translacija

2) $\zeta, D_1, D_2, u_0, v_0 = 0$; $\delta \neq 0$

2a) $\delta > 0 \Rightarrow u = \frac{1}{2}\delta x$
$v = \frac{1}{2}\delta y$

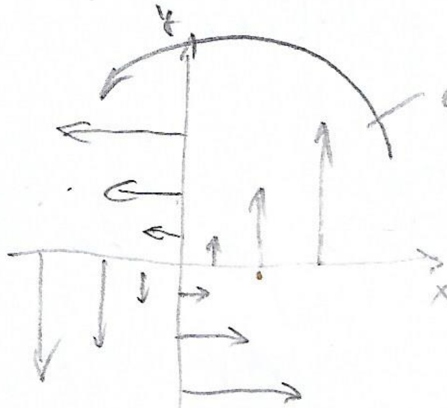


DIVERGENCIJA

2b) $\delta < 0 \Rightarrow \overline{DZ}$ $\rightarrow$ KONVERGENCIJA

3) $\delta, D_1, D_2, u_0, v_0 = 0$; $\zeta \neq 0$

3a) $\zeta > 0 \Rightarrow u = -\frac{1}{2}\zeta y$
$v = \frac{1}{2}\zeta x$



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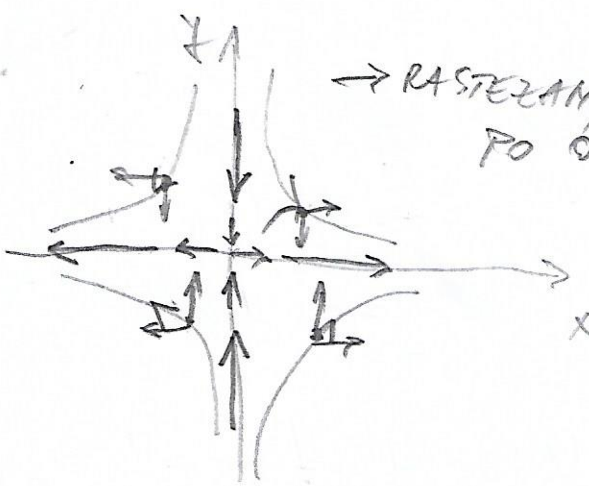
3b) $\zeta < 0 \Rightarrow \overline{DZ} \Rightarrow$ AC vrtl.

4) $\delta, \zeta, D_2, u_0, v_0 = 0$; $D_1 \neq 0$

4a) $D_1 > 0 \Rightarrow u = \frac{1}{2} D_1 x$
$v = -\frac{1}{2} D_1 y$



$\rightarrow$ RASTEZANJE PO OSI X

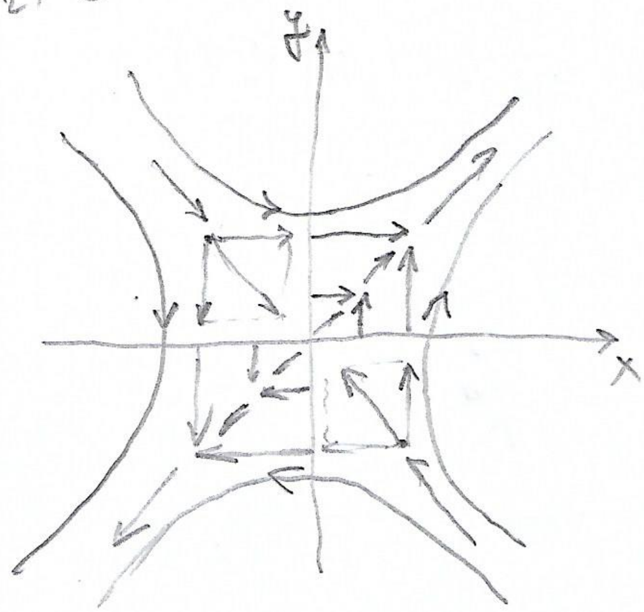
4b) $D_1 < 0 \Rightarrow \overline{DZ}$ rastezanje po osi y

(5) $\delta, \xi, D_1, u_0, v_0 = 0$; $D_2 \neq 0$

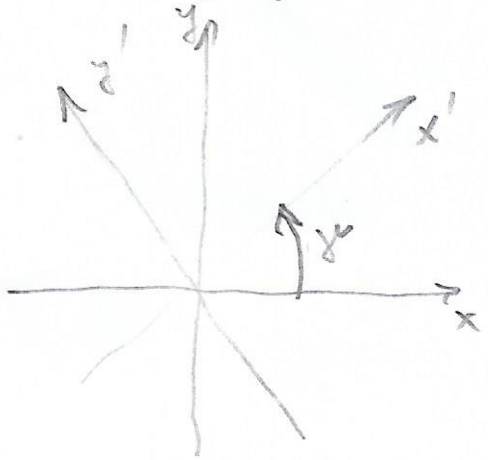
5a) $D_2 > 0 \Rightarrow u = \frac{1}{2} D_2 y$

$v = \frac{1}{2} D_2 x$

→ to je u biti D_1 zarotiran za 45°!



– u praksi je najbolji izbor rotiranog sustava u kojem vrijedi $D_2' = 0$



$$x' = x\cos\gamma + y\sin\gamma$$
$$y' = -x\sin\gamma + y\cos\gamma$$

– vrijedi: $\delta' = \delta$

$\xi' = \xi$

DZ POKAŽI $D_1' = D_1 \cos 2\gamma + D_2 \sin 2\gamma$

$D_2' = -D_1 \sin 2\gamma + D_2 \cos 2\gamma$

$D_2' = 0 \Rightarrow$ (1) $D_1' = D_1 \cos 2\gamma_d + D_2 \sin 2\gamma_d$

(2) $0 = -D_1 \sin 2\gamma_d + D_2 \cos 2\gamma_d$

$$\Rightarrow D_1' = \frac{D_1}{\cos 2\gamma_d} = \frac{D_2}{\sin 2\gamma_d} = D$$

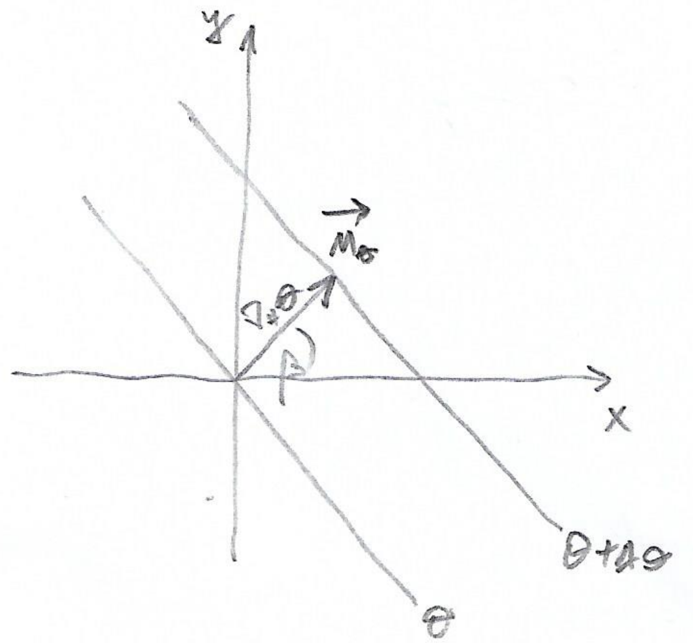
$$\Rightarrow u' = u_0' + \frac{1}{2}\delta x' - \frac{1}{2}\xi y' + \frac{1}{2} D x'$$
$$v' = v_0' + \frac{1}{2}\delta y' + \frac{1}{2}\xi x' - \frac{1}{2} D y'$$

$$\frac{\partial u'}{\partial x} = \frac{1}{2}(\delta + D) \qquad \frac{\partial v'}{\partial x} = \frac{1}{2}\xi$$

$$\frac{\partial u'}{\partial y} = -\frac{1}{2}\xi \qquad \frac{\partial v'}{\partial y} = \frac{1}{2}(\delta - D)$$

- horizontalna ravnina ⇒ (2) prelazi u (uz pp. adijabatičnosti):

$$F = -\vec{n}_\theta \cdot \left(\frac{\partial \theta}{\partial x} \nabla_H u + \frac{\partial \theta}{\partial y} \nabla_H v \right)$$



$$\vec{n}_\theta = \cos\beta \, \vec{i} + \sin\beta \, \vec{j}$$

$$\frac{\partial \theta}{\partial x} = |\nabla_H \theta| \cos\beta$$

$$\frac{\partial \theta}{\partial y} = |\nabla_H \theta| \sin\beta$$

$$F = -(\cos\beta \, \vec{i} + \sin\beta \, \vec{j}) \Big[|\nabla_H \theta| \cos\beta \left(\frac{\partial u}{\partial x} \vec{i} + \frac{\partial u}{\partial y} \vec{j} \right) + |\nabla_H \theta| \sin\beta \left(\frac{\partial v}{\partial x} \vec{i} + \frac{\partial v}{\partial y} \vec{j} \right) \Big]$$

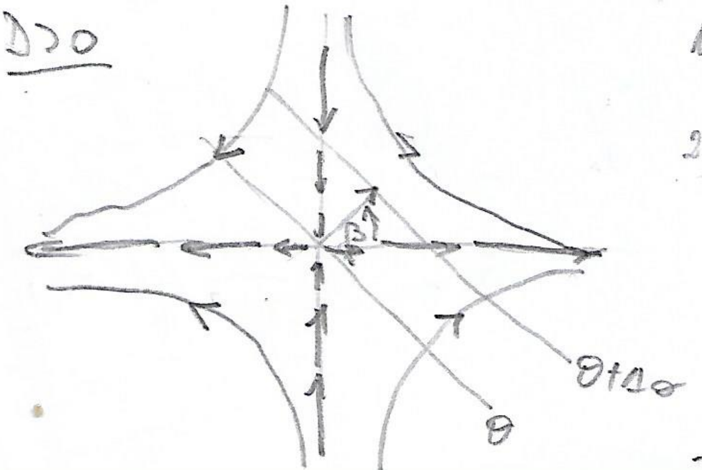
- uz $D_2 = 0$ ⇒ $\frac{\partial u}{\partial x} = \frac{1}{2}(\delta + D)$ $\quad \frac{\partial v}{\partial x} = \frac{1}{2}\zeta$

$\frac{\partial u}{\partial y} = -\frac{1}{2}\zeta$ $\quad \frac{\partial v}{\partial y} = \frac{1}{2}(\delta - D)$

$$\Rightarrow \boxed{F = -\frac{1}{2} |\nabla_H \theta| (D \cos 2\beta + \delta)} \quad 2(3).$$

$\delta > 0$ ⇒ divergencija ⇒ FL
$\delta < 0$ ⇒ konvergencija ⇒ FG

D>0



1) $\beta \in [0 - 45°]$ ⇒ $\cos 2\beta > 0$ ⇒ $F < 0$ ⇒ FL

2) $\beta \in [45 - 90°]$ ⇒ $\cos 2\beta < 0$ ⇒ $F > 0$ ⇒ FG

$D < 0$ ⇒ DZ