

# 9. DVOSTRUKI I TROSTRUKI INTEGRAL

## 9.1. Izračunajte :

$$(a) \iint_{\Omega} e^{x+y} dx dy, \quad \Omega = \{(x, y) \in \mathbb{R}^2 : 0 \leq x \leq 1, 0 \leq y \leq 3\},$$

$$(b) \iint_{\Omega} x^3 y dx dy, \quad \Omega = \{(x, y) \in \mathbb{R}^2 : 0 \leq x \leq 1, 0 \leq y \leq x\},$$

$$(c) \iint_{\Omega} x^2 y^2 dx dy, \quad \Omega = \{(x, y) \in \mathbb{R}^2 : 0 \leq x \leq 1, 0 \leq y \leq x\},$$

$$(d) \iint_{\Omega} \cos(x+y) dx dy, \quad \Omega = \{(x, y) \in \mathbb{R}^2 : 0 \leq x \leq \frac{\pi}{2}, 0 \leq y \leq \frac{\pi}{2}\},$$

$$(e) \iint_{\Omega} (x+y^3) dx dy, \quad \Omega = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 1\},$$

$$(f) \iint_{\Omega} ye^x dx dy, \quad \Omega = \{(x, y) \in \mathbb{R}^2 : 0 \leq y \leq 1, 0 \leq x \leq y^2\},$$

$$(g) \iint_{\Omega} (x^4 + y^2) dx dy, \quad \Omega \text{ skup omeđen sa } y = x^3 \text{ i } y = x^2,$$

$$(h) \iint_{\Omega} e^{-\frac{y^2}{2}} dx dy, \quad \Omega \text{ skup omeđen sa } x = 0, 2y = x, y = 1,$$

$$(i) \iint_{\Omega} (x+y) dx dy, \quad \Omega \text{ skup omeđen sa } y = x^4 \text{ i } y = x^3,$$

$$(j) \iint_{\Omega} \sqrt{xy} dx dy, \quad \Omega = \{(x, y) \in \mathbb{R}^2 : 0 \leq y \leq 1, y^2 \leq x \leq y\},$$

- (k)  $\iint_{\Omega} (4 - y^2) dx dy$ ,  $\Omega$  skup omeđen sa  $y^2 = 2x$  i  $y^2 = 8 - 2x$ ,
- (l)  $\iint_{\Omega} e^{x^2} dx dy$ ,  $\Omega$  skup omeđen sa  $y = 0$ ,  $2y = x$  i  $x = 2$ ,
- (m)  $\iint_{\Omega} (3xy^3 - y) dx dy$ ,  $\Omega$  skup omeđen sa  $y = |x|$  i  $y = -|x|$  za  $x \in [-1, 1]$ .

9.2. Odredite površinu skupa omeđenog sa:

- (a)  $y = x$  i  $x = 4y - y^2$ ,
- (b)  $x + y = 5$  i  $xy = 6$ ,
- (c)  $x^2 = 4y$  i  $2y - x - 4 = 0$ ,
- (d)  $y = x$  i  $4y^3 = x^2$ ,
- (e)  $xy = 2$ ,  $y = 1$  i  $y = x + 1$ .

9.3. Promijenite redoslijed integracije i izračunajte:

- (a)  $\int_{-1}^0 \int_{-\sqrt{y+1}}^{\sqrt{y+1}} x^2 dx dy$ ,
- (b)  $\int_0^1 \int_{x^2}^1 \frac{x^3}{\sqrt{x^4+y^2}} dy dx$ ,
- (c)  $\int_0^1 \int_{\sqrt{x}}^1 \sin\left(\frac{y^3+1}{2}\right) dy dx$ ,

(d)  $\int_1^2 \int_0^{\ln y} e^{-x} dx dy.$

- 9.4. Odredite volumen tijela omeđenog s gornje strane ravninom  $z = x + y$ , a s donje strane trokutom s vrhovima  $(0, 0)$ ,  $(0, 1)$  i  $(1, 0)$ .
- 9.5. Odredite volumen tijela omeđenog s gornje strane ravninom  $z = 2x + 3y$ , a s donje strane  $0 \leq x \leq 1$ ,  $0 \leq y \leq 1$ .
- 9.6. Odredite volumen tijela omeđenog sa  $\frac{x}{2} + \frac{y}{3} + \frac{z}{4} = 1$  i koordinatnim ravninama.
- 9.7. Odredite volumen tijela omeđenog sa  $x = 0$ ,  $y = 0$ ,  $z = 0$ ,  $x + y = 4$  i  $z = x + y + 1$ .
- 9.8. Odredite volumen tijela omeđenog s gornje strane plohom  $z = 1 + xy$ , a s donje strane trokutom s vrhovima  $(1, 1)$ ,  $(4, 1)$  i  $(3, 2)$ .
- 9.9. Odredite volumen tetraedra omeđenog koordinatnim ravninama i ravninom  $x + y + z = 1$ .

## 9.10. Izračunajte :

$$(a) \int_0^2 \int_0^x \int_0^y y \, dz dy dx,$$

$$(b) \int_0^1 \int_1^{2y} \int_0^x (x + 2z) \, dz dx dy,$$

$$(c) \int_0^1 \int_{1-x}^{1+x} \int_0^{xy} 4z \, dz dy dx,$$

$$(d) \int_0^2 \int_{-1}^1 \int_1^3 (z - xy) \, dy dx dz,$$

$$(e) \int_0^{\frac{\pi}{2}} \int_0^1 \int_0^{\sqrt{1-x^2}} x \cos z \, dy dx dz,$$

$$(f) \int_{-1}^2 \int_1^{y-2} \int_e^{e^2} \frac{x+y}{z} \, dz dx dy,$$

$$(g) \int_1^2 \int_y^{y^2} \int_0^{\ln x} ye^z \, dz dx dy,$$

$$(h) \int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} \int_0^1 e^z \cos x \sin y \, dz dy dx.$$

# Rješenja:

9.1. (a)  $e^4 - e^3 - e + 1,$

(b)  $\frac{1}{12},$

(c)  $\frac{1}{18},$

(d)  $0,$

(e)  $0,$

(f)  $\frac{1}{2}(e - 2),$

(g)  $\frac{9}{280},$

(h)  $2\left(1 - \frac{1}{\sqrt{e}}\right),$

(i)  $\frac{31}{630},$

(j)  $\frac{2}{27},$

(k)  $\frac{512}{15},$

(l)  $\frac{1}{4}(e^4 - 1),$

(m)  $0.$

9.2. (a)  $\frac{9}{2},$

(b)  $\frac{5}{2} + 6 \ln \frac{2}{3},$

(c)  $9,$

(d)  $\frac{1}{160},$

(e)  $\ln 4 - \frac{1}{2}.$

- 9.3. (a)  $\frac{4}{15}$ ,  
(b)  $\frac{1}{4}(\sqrt{2} - 1)$ ,  
(c)  $\frac{2}{3}(\cos \frac{1}{2} - \cos 1)$ ,  
(d)  $1 - \ln 2$ .

9.4.  $\frac{1}{3}$ .

9.5.  $\frac{5}{2}$ .

9.6. 4.

9.7.  $\frac{88}{3}$ .

9.8.  $\frac{55}{8}$ .

9.9.  $\frac{1}{6}$ .

- 9.10. (a)  $\frac{4}{33}$ ,  
(b)  $\frac{3}{33}$ ,  
(c)  $\frac{11}{9}$ ,  
(d) 8,  
(e)  $\frac{1}{33}$ ,  
(f)  $\frac{3}{33}$ ,  
(g)  $\frac{47}{24}$ ,  
(h)  $e - 1$ .