has been applied to spectra to help organize the spectral results, and to help focus our understanding about turbulence.

As discussed in the previous chapter, the discrete power spectral intensity measures how much of the variance of a signal is associated with a particular frequency, f. If ξ represents any variable, then the discrete power spectral intensity $E_\xi(f)$ has units of ξ^2. An obvious way to make the spectral intensity dimensionless is to divide it by the total variance $\overline{\xi'^2}$. A continuous spectrum with power spectral density of $S_\xi(f)$ has the same units as ξ^2/f, and can be made dimensionless by dividing by $\overline{\xi'^2}/f$. Analogous expressions can be made for wavenumber spectra instead of frequency spectra. In both of these cases, the result is a spectrum that gives the fraction of total variance explained by a wavelength or wavelength band.

Alternately, if the turbulence is driven or governed by specific mechanisms, such as wind shear, buoyancy, or dissipation, then the spectral intensities can be normalized by scaling variables appropriate to the flow. The next three Sections show normalized spectra for the inertial subrange, for surface layer turbulence generated mechanically, and for mixed layer turbulence generated buoyantly.

9.9.1 Inertial Subrange

See p. 430

As discussed in Chapter 5, there are many situations where middle size turbulent eddies "feel" neither the effects of viscosity, nor the generation of TKE. These eddies get their energy inertially from the larger-size eddies, and lose their energy the same way to smaller-size eddies. For a steady-state turbulent flow, the cascade rate of energy down the spectrum must balance the dissipation rate at the smallest eddy sizes. Hence, there are only three variables relevant to the flow: S, κ, and ε. This similarity approach was pioneered by Kolmogorov (1941) and Obukhov (1941).

By performing a Buckingham Pi dimensional analysis, we can make only one dimensionless group from these three variables:

$$\pi_1 = \frac{S^3\,\kappa^5}{\varepsilon^2}$$

We know that this Pi group must be equal to a constant, because there are no other Pi groups for it to be a function of.

Solving the above equation for S yields:

$$S(\kappa) = \alpha_k\,\varepsilon^{2/3}\,\kappa^{-5/3} \qquad (9.9.1)$$

where the α_k is known as the ***Kolmogorov constant***. The value of this constant has yet to be pinned down (Gossard, et.al., 1982), but it is in the range of $\alpha_k = 1.53$ to 1.68.

$$\Delta E_k \sim \varepsilon^{\alpha} k^{\beta}$$

$$\left.\frac{m^2 s^{-2}}{m^{-1}} \sim (m^2 s^{-3})^{\alpha}(m^{-1})^{\beta}\right\} \Rightarrow \begin{array}{l} 3 = 2\alpha - \beta \\ -2 = -3\alpha \Rightarrow \alpha = \frac{2}{3} \\ \beta = 2\cdot\frac{2}{3} - 3 = -\frac{5}{3} \end{array}$$

One of the easiest ways to determine whether any measured spectrum has an inertial subrange is to plot the spectrum (S vs. κ) on a log-log graph. The inertial subrange portion should appear as a straight line with a -5/3 slope (see Fig 9.12). The demonstration spectra plotted in Fig 8.9 all have an inertial subrange at normalized frequencies greater than 2.5, assuming that Taylor's hypothesis can be used to relate frequencies to wavenumbers via $f = \overline{M}\cdot\kappa$.

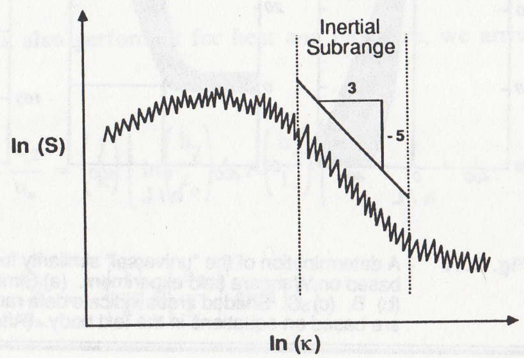

Fig. 9.12
On a log-log spectral plot, the inertial subrange appears as a straight line with -5/3 slope.

9.9.2 Surface Layer Spectra

Suppose that the velocity spectra $fS_u(f)$ for a surface layer in a state of forced convection were likely to be affected by the following variables: u_*, $\overline{w'\theta_v'}_s$, z, $\overline{U}$ (or $\overline{M}$), f, and ε. Buckingham Pi analysis of the above variables gives three dimensionless groups: $\pi_1 = f\,S_u(f) / (k\,z\,\varepsilon)^{2/3}$, $\pi_2 = f\,z / \overline{M}$, and $\pi_3 = z / L$.

Fig 9.13a shows the result when these π groups are plotted (Kaimal, et al, 1972). We see some important characteristics: (1) The peak spectral intensity is reduced as the static stability is increased, because stability is opposing turbulent motions. (2) The peak is shifted to higher frequencies as stability is increased, possibly because the lower frequencies are more strongly damped by the buoyancy forces. (3) At high frequencies, the spectral intensity is no longer dependent on the static stability (at least for the weak stabilities plotted), suggesting that the smaller size eddies in the inertial subrange receive all of their energy via the cascade process from larger eddies, with no direct interaction with the mean flow or the mean stratification. (4) Finally, there is a curious occurrence of an ***excluded region*** in the spectral plot near neutral stratification (lightly shaded in the figure).

$$\Delta E_k = S(k) = \text{const}\;\varepsilon^{2/3}\,k^{-5/3}$$