Development of a Turbulence Closure Model for Geophysical Fluid Problems

G. L. Mellor and T. Yamada

Reviews of Geophysics and Space Physics Vol. 20, No. 4, 851-875, 1982.

4.2.2016.

Andreina Belušić

Sadržaj

- Motivacija
- Reynolds usrednjene N-S jednadžbe
- M-Y model nivo 4, 3
- M-Y model nivo 2.5, 2
- Određivanje empiričkih konstanti i RU
- Primjena na geofizičke fluide

Motivacija i Cilj

- U manje od desetljeća dogodio se napredak u modeliranju; Iz modeliranja samo srednjeg stanja do potrebe da se modeliraju i procjene varijance i kovarijance turbulentnih polja.
- Sintetizirati i organizirati znanja koja su se do 1982. godine primjenjivala o redovima zatvaranja jednadžbi te dodati nove, korisne spoznaje na tu temu.

Reynolds usrednjene N-S jednadžbe

$$\frac{\partial \overline{\rho}}{\partial t} + \frac{\partial(\overline{\rho}\,\overline{U_i})}{\partial x_i} = 0 \tag{1}$$

$$\overline{\rho}\left(\frac{\partial \overline{U_j}}{\partial t} + \overline{U_k}\frac{\partial \overline{U_j}}{\partial x_k}\right) + \overline{\rho}\epsilon_{jkl} f_k \overline{U_l} = \frac{\partial(\overline{\rho}\,\overline{u'_k u'_j})}{\partial x_k} - \frac{\partial \overline{P}}{\partial x_j} - g_j \overline{\rho} \tag{2}$$

$$\overline{\rho}\left(\frac{\partial \overline{\Theta}}{\partial t} + \overline{U_k}\frac{\partial \overline{\Theta}}{\partial x_k}\right) = \frac{\partial(\overline{\rho}\,\overline{u'_k \theta'})}{\partial x_k} \tag{3}$$

Osnovne jednadžbe

- Zatvaranje sustava:
  1.) Hipoteza o redistribuciji energije (Rotta; 1951.)

$$\overline{\frac{p'}{\overline{\rho}}\left(\frac{\partial u_i'}{\partial x_j}+\frac{\partial u_j'}{\partial x_i}\right)} = -\frac{\sqrt{\overline{u_i'^2}}}{3l_1}\left(\overline{u_i'u_j'}-\frac{\delta_{ij}}{3}\overline{u_i'^2}\right)+C_1\overline{u_i'^2}\left(\frac{\partial \overline{U_i}}{\partial x_j}+\frac{\partial \overline{U_j}}{\partial x_i}\right) \quad (4)$$

  2.) Hipoteza lokalne izotropije male skale (Kolmogorov; 1942.)

$$2\nu\overline{\frac{\partial u_i'}{\partial x_k}\frac{\partial u_j'}{\partial x_k}} = \frac{2}{3}\frac{\overline{u_i'^2}^{3/2}}{\Lambda_1}\delta_{ij} \qquad \overline{\frac{p'}{\overline{\rho}}\frac{\partial \theta'}{\partial x_j}} = -\frac{\sqrt{\overline{u_i'^2}}}{3l_2}\overline{u_j'\theta'} \quad (5)$$

$$(\alpha+\nu)\left(\overline{\frac{\partial u_j'}{\partial x_k}\frac{\partial \theta'}{\partial x_k}}\right) = 0 \qquad 2\alpha\left(\overline{\frac{\partial \theta'}{\partial x_k}\frac{\partial \theta'}{\partial x_k}}\right) = 2\frac{\sqrt{\overline{u_i'^2}}}{\Lambda_2}\overline{\theta'^2} \quad (6)$$

  3.)

$$\overline{u_i'u_j'u_k'} = \frac{3}{5}l\sqrt{\overline{u_i'^2}}S_u\left(\frac{\partial\overline{u_i'u_j'}}{\partial x_k}+\frac{\partial\overline{u_i'u_k'}}{\partial x_j}+\frac{\partial\overline{u_j'u_k'}}{\partial x_i}\right) \qquad \overline{u_k'u_j'\theta'} = l\sqrt{\overline{u_i'^2}}S_{u\theta}\left(\frac{\partial\overline{u_k'\theta'}}{\partial x_j}+\frac{\partial\overline{u_j'\theta'}}{\partial x_k}\right) \quad (7)$$

$$\overline{p'u_i'} = l\sqrt{\overline{u_i'^2}}S_u'\frac{\partial\overline{u_i'^2}}{\partial x_i} = 0 \qquad \overline{u_k'\theta'\theta'} = -l\sqrt{\overline{u_i'^2}}S_\theta\frac{\partial\overline{\theta'^2}}{\partial x_k} \qquad \overline{p'\theta'} = 0 \quad (8)$$

Osnovne jednadžbe

- Najveći nedostatak modela je tzv.'glavna skala duljine l'

$$(l_1, \Lambda_1, l_2, \Lambda_2) = (A_1, B_1, A_2, B_2,)l \tag{9}$$

- A_1, B_1, A_2, B_2, C_1 se određuju iz podataka kada vrijedi da je produkcija TKE jednaka disipaciji TKE.

- Ostali nepoznati bezdimenzionalni parametri: $S_u, S_{u\theta}, S_\theta$.

M-Y model nivo 4

- Osnovne jednadžbe + jednadžbe za prognozu momenata 2. reda (uz Rotta i Kolmogorov pretpostavke) + $S_u = S_\theta = S_{u\theta} \rightarrow$ zatvoreni sustav

$$\frac{\partial \overline{u_i' u_j'}}{\partial t} + \overline{U_k} \frac{\partial \overline{u_i' u_j'}}{\partial x_k} + \frac{\partial}{\partial x_k}[\frac{3}{5} l \sqrt{\overline{u_i'^2}} S_u (\frac{\partial \overline{u_i' u_j'}}{\partial x_k} + \frac{\partial \overline{u_i' u_k'}}{\partial x_j} + \frac{\partial \overline{u_j' u_k'}}{\partial x_i})] = -\frac{\sqrt{\overline{u_i'^2}}}{3 l_1} (\overline{u_i' u_j'} - \frac{\delta_{ij}}{3} \overline{u_i'^2}) + C_1 \overline{u_i'^2} (\frac{\partial \overline{U_i}}{\partial x_j} + \frac{\partial \overline{U_j}}{\partial x_i})$$
$$- \frac{2}{3} \frac{\overline{u_i'^2}^{3/2}}{\Lambda_1} \delta_{ij} - \overline{u_k' u_i'} \frac{\partial \overline{U_j}}{\partial x_k} - \overline{u_k' u_j'} \frac{\partial \overline{U_i}}{\partial x_k} + \frac{1}{\overline{\Theta}} (g_j \overline{u_i' \theta'} + g_i \overline{u_j' \theta'}) - f_k (\epsilon_{jkl} \overline{u_i' u_l'} + \epsilon_{ikl} \overline{u_j' u_l'}) \qquad (10)$$

$$\frac{\partial \overline{u_j' \theta'}}{\partial t} + \overline{U_k} \frac{\partial \overline{u_j' \theta'}}{\partial x_k} - \frac{\partial}{\partial x_k}[l \sqrt{\overline{u_i'^2}} S_{u\theta} (\frac{\partial \overline{u_j' \theta'}}{\partial x_k} + \frac{\partial \overline{u_k' \theta'}}{\partial x_j}] = -\overline{u_k' u_j'} \frac{\partial \overline{\Theta}}{\partial x_k} - \overline{u_k' \theta'} \frac{\partial \overline{U_i}}{\partial x_k} + \frac{1}{\overline{\Theta}} g_j \overline{\theta' \theta'} - \frac{\sqrt{\overline{u_i'^2}}}{3 l_2}$$
$$\overline{u_j' \theta'} - f_k \epsilon_{jkl} \overline{u_l' \theta'} \qquad (11)$$

$$\frac{\partial \overline{\theta'^2}}{\partial t} + \overline{U_k} \frac{\partial \overline{\theta'^2}}{\partial x_k} - \frac{\partial}{\partial x_k}[l \sqrt{\overline{u_i'^2}} S_\theta \frac{\partial \overline{\theta'^2}}{\partial x_k}] = -2 \overline{u_k' \theta'} \frac{\partial \overline{\Theta}}{\partial x_k} - 2 \frac{\sqrt{\overline{u_i'^2}} \overline{\theta^2}}{\Lambda_2} \qquad (12)$$

M-Y model nivo 3

- Osnovne jednadžbe + prognoza TKE + prognoza za varijancu potencijalne temeperature + dijagnozaostalih momenata $\longrightarrow$ zatvoreni sustav

$$\frac{\partial \overline{u_i'^2}}{\partial t} + \overline{U_k}\frac{\partial \overline{u_i'^2}}{\partial x_k} + \frac{\partial}{\partial x_k}[l\sqrt{\overline{u_i'^2}}S_u \frac{\partial \overline{u_i'^2}}{\partial x_k}] = -2\overline{u_j' u_i'}\frac{\partial \overline{U_i}}{\partial x_j} - 2\frac{1}{\overline{\Theta}} g_i \overline{u_i'\theta'} - 2\frac{\overline{u_i'^2}^{3/2}}{\Lambda_1} \quad (13)$$

$$\frac{\partial \overline{\theta'^2}}{\partial t} + \overline{U_k}\frac{\partial \overline{\theta'^2}}{\partial x_k} - \frac{\partial}{\partial x_k}[l\sqrt{\overline{u_i'^2}}S_\theta (\frac{\partial \overline{\theta'^2}}{\partial x_k})] = -2\overline{u_k'\theta'}\frac{\partial \overline{\Theta}}{\partial x_k} - 2\frac{\sqrt{\overline{u_i'^2}}\overline{\theta'^2}}{\Lambda_2} \quad (14)$$

$$\overline{u_i' u_j'} = \frac{\delta_{ij}}{3}\overline{u_i'^2} - \frac{3l_1}{\sqrt{\overline{u_i'^2}}}[\overline{u_k' u_i'}\frac{\partial \overline{U_j}}{\partial x_k} - \overline{u_k' u_j'}\frac{\partial \overline{U_i}}{\partial x_k} - \frac{2}{3}\delta_{ij}\overline{u_i' u_j'}]\frac{\partial \overline{U_i}}{\partial x_j} - C_1\overline{u_i'^2}(\frac{\partial \overline{U_i}}{\partial x_j} + \frac{\partial \overline{U_j}}{\partial x_i}) + \frac{1}{\overline{\Theta}}(g_j\overline{u_i'\theta'} + g_i\overline{u_j'\theta'}) + \frac{2}{3}\delta_{ij}\frac{1}{\overline{\Theta}}(g_i\overline{u_i'\theta'}) \quad (15)$$

$$\overline{u_j'\theta'} = -\frac{3l_2}{\sqrt{\overline{u_i'^2}}}[\overline{u_k' u_j'}\frac{\partial \overline{\Theta}}{\partial x_k} - \overline{u_k'\theta'}\frac{\partial \overline{U_j}}{\partial x_k} + \frac{1}{\overline{\Theta}}(g_j\overline{\theta'\theta'} + f_k\epsilon_{jkl}\overline{u_l'\theta'}] \quad (16)$$

M-Y model nivo 2.5

- Zanemarivanje totalnih derivacija po vremenu i difuzije u jednadžbi (14)

$$\frac{\partial \overline{u_i'^2}}{\partial t} + \overline{U_k}\frac{\partial \overline{u_i'^2}}{\partial x_k} + \frac{\partial}{\partial x_k}[l\sqrt{\overline{u_i'^2}}S_u\frac{\partial \overline{u_i'^2}}{\partial x_k}] = -2\overline{u_j'u_i'}\frac{\partial \overline{U_i}}{\partial x_j} - 2\frac{1}{\overline{\Theta}}g_i\overline{u_i'\theta'} - 2\frac{\overline{u_i'^2}^{3/2}}{\Lambda_1} \qquad (17)$$

$$0 = -2\overline{u_k'\theta'}\frac{\partial \overline{\Theta}}{\partial x_k} - 2\frac{\sqrt{\overline{u_i'^2}}\,\overline{\theta'^2}}{\Lambda_2} \rightarrow \overline{\theta'^2} = -\frac{\Lambda_2}{\sqrt{\overline{u_i'^2}}}\overline{u_k'\theta'}\frac{\partial \overline{\Theta}}{\partial x_k} \qquad (18)$$

- Osnovne jednadžbe + prognoza za TKE + dijagnoza ostalih momenata drugog reda + jednadžba za glavnu turbulentnu skalu duljine $\longrightarrow$ zatvoreni sustav

$$\frac{\partial \overline{u_i'^2}l}{\partial t} + \overline{U_k}\frac{\partial \overline{u_i'^2}l}{\partial x_k} - \frac{\partial}{\partial z}[\sqrt{\overline{u_i'^2}}lS_u\frac{\partial (\overline{u_i'^2}l)}{\partial z}] = lE_1[-\overline{u_i'u_j'}\frac{\partial \overline{U_i}}{\partial x_j} - \frac{1}{\overline{\Theta}g_i\overline{u_i'\theta'}}] - \frac{\sqrt{u_i'}^{3/2}}{B_1}[1+E_2(\frac{l}{kL})^2] \qquad (19)$$

M-Y model nivo 2

- Zanemarivanje svih totalnih derivacija po vremenu i difuzije. (13) tada postaje ravnoteža produkcije i disipacije u TKE:

$$0 = -\overline{u_j' u_i'} \frac{\partial \overline{U_i}}{\partial x_j} - \frac{1}{\overline{\Theta}} g_i \overline{u_i' \theta'} - \frac{\overline{u_i'^2}^{3/2}}{\Lambda_1} \tag{20}$$

$$\overline{u_i' u_j'} = \frac{\delta_{ij}}{3} \overline{u_i'^2} - \frac{3l_1}{\sqrt{\overline{u_i'^2}}} [\overline{u_k' u_i'} \frac{\partial \overline{U_j}}{\partial x_k} - \overline{u_k' u_j'} \frac{\partial \overline{U_i}}{\partial x_k} - \frac{2}{3} \delta_{ij} \overline{u_i' u_j'}] \frac{\partial \overline{U_i}}{\partial x_j} - C_1 \overline{u_i'^2} (\frac{\partial \overline{U_i}}{\partial x_j} + \frac{\partial \overline{U_j}}{\partial x_i}) + \frac{1}{\overline{\Theta}} (g_j \overline{u_i' \theta'} + g_i \overline{u_j' \theta'}) + \frac{2}{3} \delta_{ij} \frac{1}{\overline{\Theta}} (g_i \overline{u_i' \theta'}) \tag{21}$$

$$\overline{u_j' \theta'} = -\frac{3l_2}{\sqrt{\overline{u_i'^2}}} [\overline{u_k' u_j'} \frac{\partial \overline{\Theta}}{\partial x_k} - \overline{u_k' \theta'} \frac{\partial \overline{U_j}}{\partial x_k} + \frac{1}{\overline{\Theta}} (g_j \overline{\theta' \theta'} + f_k \epsilon_{jkl} \overline{u_l' \theta'}] \tag{22}$$

$$\overline{\theta'^2} = -\frac{\Lambda_2}{\sqrt{\overline{u_i'^2}}} \overline{u_k' \theta'} \frac{\partial \overline{\Theta}}{\partial x_k} \tag{23}$$

- Zatvoreni sustav 4 jednadžbe s 4 nepoznanice

M-Y model nivo 2.5 i 2

Uvođenje aproksimacije atmosferskog graničnog sloja

- Zanemarivanje Coriolisovog člana u jednadžbi za turbulentne tokove
- H.H i $\overline{w}$=0

$$\overline{\rho}\frac{\partial\overline{U}}{\partial t}+\frac{\partial}{\partial z}[\overline{\rho}\overline{u'w'}]=-\frac{\partial\overline{P}}{\partial x}+\overline{\rho}f\overline{V} \qquad \overline{\rho}\frac{\partial\overline{V}}{\partial t}+\frac{\partial}{\partial z}[\overline{\rho}\overline{v'w'}]=-\frac{\partial\overline{P}}{\partial y}-\overline{\rho}f\overline{U} \tag{24}$$

$$0=-\frac{\partial\overline{P}}{\partial z}-\overline{\rho}g \qquad \frac{\partial\overline{\Theta}}{\partial t}+\frac{\partial(\overline{w'\theta'})}{\partial z}=0 \tag{25}$$

- K-teorija

$$\overline{u'w'}=-K_M\frac{\partial\overline{U}}{\partial z} \qquad \overline{v'w'}=-K_M\frac{\partial\overline{V}}{\partial z} \qquad \overline{w'\theta'}=-K_H\frac{\partial\overline{\Theta}}{\partial z} \tag{26}$$

Određivanje konstanti A1, B1, A2, B2, C1, S_u, E1, E2

- Bazira se na podacima za neutralni AGS

  B1 goes to Eps [here = 15, MIUU model later =~ 22]

- A1, B1, A2 , B2 , C1 $\rightarrow$ 0.78, 15.0, 0.79, 8.0, 0.23 [Mellor, 1973];

- A1, B1, A2 , B2 , C1, S_u, E1, E2 $\rightarrow$ 0.92, 16.6, 0.74, 10.1, 0.08, 0.2, 1.8, 1.33 [Mellor & Yamada, 1982];

Određivanje *glavne turbulentne skale duljine l*

- 1.)

$$l = l_0 \frac{kz}{kz + l_0}, l_0 \approx \frac{\int_0^\infty z\sqrt{\overline{u_i'^2}}dz}{\int_0^\infty \sqrt{\overline{u_i'^2}}dz} \tag{27}$$

- 2.)

$$l = l_0 \tag{28}$$

Rubni uvjeti

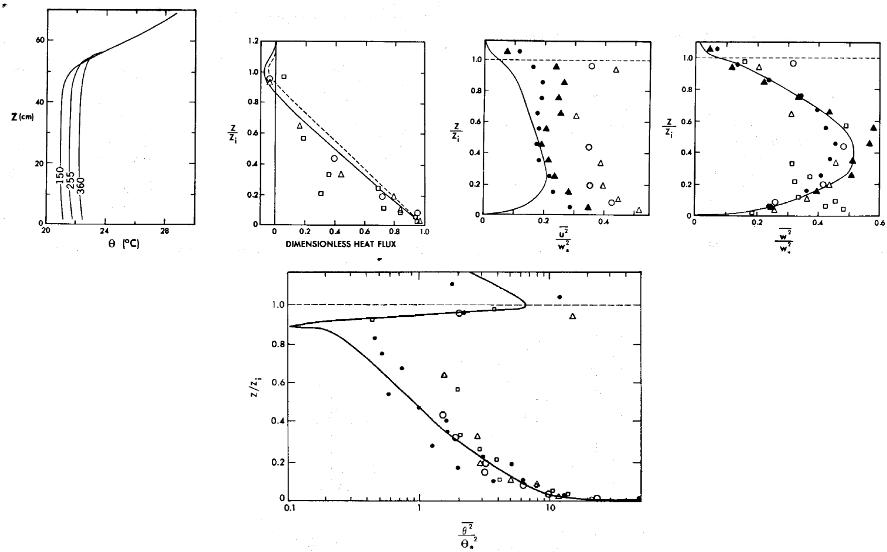
$$U(z \to z_i) \to U_g$$
$$V(z \to z_i) \to V_g$$
$$\overline{w'u'}(z \to z_i) \to 0 \qquad -\overline{w'u'}(z \to z_0) = u_*^2 cos(\alpha)$$
$$\overline{w'v'}(z \to z_i) \to 0 \qquad -\overline{w'v'}(z \to z_0) = u_*^2 sin(\alpha)$$
$$-\overline{w'\theta'}(z \to z_0) = konst.$$

$$\overline{U}(z) = \frac{u_*}{k} ln(\frac{z}{z_0})$$
$$\overline{\Theta}(z) = \overline{\Theta}(z \to z_0) + \frac{\Theta_*}{k} ln(\frac{z}{z_0})$$

Primjena na geofizičke fluide (Nivo 2.5)

Slobodna konvekcija [Wills & Deardorff, 1974.]

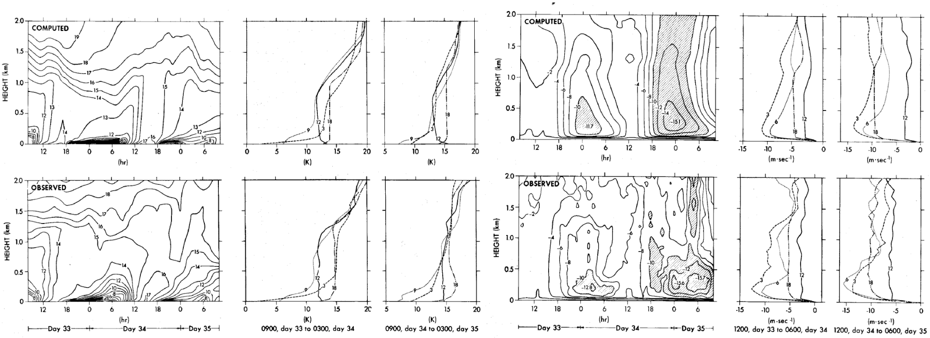
- Bez smicanja ($Ri \to \infty$)



Primjena na geofizičke fluide

Wangara eksperiment

- $l = l_0 \frac{kz}{kz + l_0}$

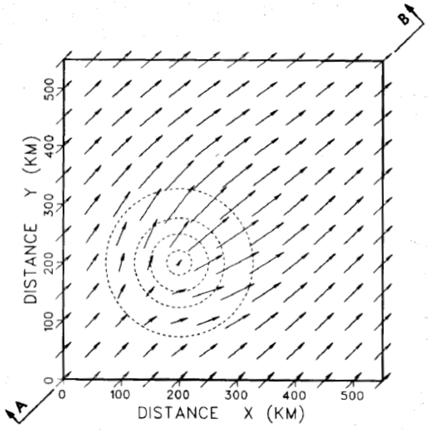


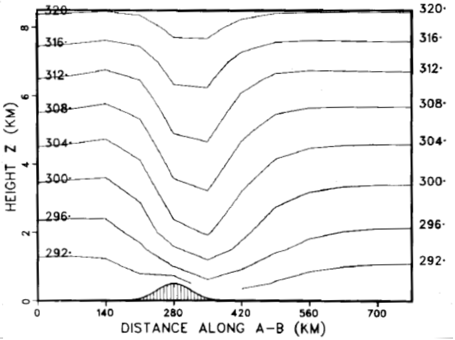
Slika: Potencijalna temperatura (K-273 K), $\overline{U}$ (m/s)

Primjena na geofizičke fluide

Strujanje preko orografije

- Gaussovska planina visine 500 m



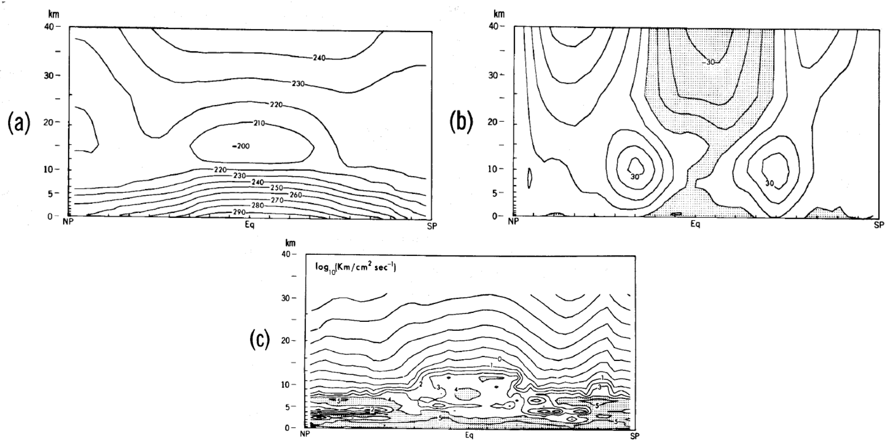


Slika: $\overline{U}$ (m/s), Potencijalna temperatura (K)

Primjena na geofizičke fluide

Opća cirkulacija atmosfere

- Rezolucija 4°, 18 vertikalnih nivoa, prvi na 2 km
- Zonalno usrednjeno



Slika: (a)Potencijalna temperatura (K), (b) $\overline{U}$ (m/s), (c) $log_{10}(K_M)$

Zaključak

- Model nadmašuje procjenu stvarnih gibanja u odnosu na sve modele prije 1973. godine
- Nivo 4 se koristi ako želimo uvažiti član povratka u izotropiju u slučaju početno anizotropnog fluida bez smicanja i uzgona
- Nivo 3 i 2.5 (koristili autori) dobro opisuju ukupni TKE
- Nivo 2.5 zahtjeva puno više računanja, ali ne daje bolje rezultate nego nivo 2
- Nivo 2 (najzastupljeniji) dobro opisuje TKE kada dominira smicanje vjetra ili uzgon, a nema disipacije

Hvala na pozornosti!